Journal of Multidisciplinary Engineering Science and Technology (JMEST)
ISSN: 3159-0040
Vol. 1 Issue 5, December - 2014

The Life Predicting Calculations in Whole Process Realized with Two kinks of Methods by means of Conventional Materials Constants under Low Cycle Fatigue Loading

Yangui Yu
Academy Office
Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering
Hangzhou, China
gx_yyg@126.com; ygyu@vip.sina.com.cn

***Abstract*—To use the theoretical approach, by means of the conventional material constants, to adopt the simple stress-, or strain-parameter way and the two-parameters multiplication method, to establish numerous new calculation models in whole damage growth process for elastic-plastic steels, which are the equations of the damage driving forces and the life predictions; To use yet different form, to put forward the damage rate linking equations and lifetime calculating expressions. For the transition value from micro to macro damage growth stage, to provide concretely the calculation processes, the steps and the methods; For some key materials constants to give New physical and geometrical significances. Thereby to realize the lifetime predicting calculations in whole process based on conventional material constants with two kinds of methods.**

Keywords—conventional material constants; micro and macro damage; simple parameter method; two-parameters multiplication method, life prediction in whole process.

I. INTRODUCTION

As everyone knows for the conventional material mechanics, that is a calculable subject, and has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the life problems for some structures when it is pre-existing flaws and under repeated loading. In that it has no to contain such calculable parameters as the damage variable D or as the crack variable a in their calculating models. On the other hand, inside the damage mechanics and the fracture mechanics, due to there are these variables, so they can just calculate above problems. But nowadays latter these disciplines are all subjects mainly depended on fatigue, damage and fracture tests.

Author thinks, in the mechanics, aviation, machinery and civil engineering etc fields, in which are also to exist such scientific principles of similar to genetic and clone technology in life science. Author has done some of works used the theoretical approach as above the similar principles [1-8]. For example, for some strength calculation models from micro to macro are provided by reference [1], for some rate calculation models from micro to macro damage growth are proposed by references [2-8], which are some models in each stage even in whole process, under different loading conditions. Two years ago, in order to do the lifetime calculations in whole process on fatigue-damage-fracture for an engineering structure, author was by means of Google Scholar to search the lifetime prediction models, as had been no found for this kind of calculation equations. After then, author continues to research this item, and bases on was provided and recently is complemented called as the comprehensive figure 1 of material fatigue-damage-fracture (or called as calculating figure of material behaviors) [3]; still applies above genetic principles, to study and analyze data in references, thereby to provide some new calculable models for the new damage growth driving force and for the lifetime predictions. Try to make the fatigue and the damage mechanics, step by step become calculable disciplines as the material mechanics, that are via theoretical calculation is given as priority, via the experiments is verified as complementary. That way, may be having practical significances for decrease experiments, stint man powers and funds, for promoting engineering applying and developing to relevant disciplines.

II. LIFE PREDICTING CALCULATIONS IN WHOLE PROCESS FOR ELASTIC-PLASTIC STEELS CONTAINING FLAW

For some elastic-plastic steels of pre-existed flaw, in micro damage growth processes, about its driving force, rate and life's calculation equations for which have been proposed in reference [1-8]; And its driving force, rate and life's calculating problems for macro damage processes, some models have also provided in references [2-3].

And in this paper, from micro-damage to macro-damage, it uses a called as "the single parameters method" and "two-parameter multiplication-method" for the life put up the whole process calculations, that are by means of the stress σ or the ε as "genetic element" in first stage [3], or by the stress intensity factor K_1 and the crack tip open displacement range δ_t as "genetic element" in the second stage, to establish various calculable models for the driving force, the rate and the fife, thereby achieve life prediction calculations in whole process under low cycle fatigue loading.

A. The Calculations for micro damage process

1) The single parameter method

Under $\sigma > \sigma_s$ condition, micro-damage life equation corresponded to reversed direction curve C_1C in Fig1, here to adopt the strain range $\Delta\varepsilon_p$ expressing, that is as following form

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{B_1^{'} \times (\Delta I)^{m'_1}} (Cycle), (\sigma > \sigma_s) \quad \text{(1-1)}$$

Or $N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{B_1^{'} \times (\Delta\varepsilon_p)^{m'_1} D_1} (Cycle), (\sigma > \sigma_s)$ (1-2)

$$I'_1 = (\varepsilon_p)^{m_1^{'}} \cdot D_1, (\%^{m_1^{'}} \cdot damage - nuit - namber) \quad (2)$$

$$\Delta I'_1 = (\Delta\varepsilon_p)^{m_1^{'}} \cdot D_1, (\%^{m_1^{'}} \cdot damage - nuit - namber) \quad (3)$$

$$B_1^{'} = 2[2\varepsilon'_f]^{-m'_1} \times (v_{eff})^{-1} \quad (4)$$

Here the damage variable D_1 (or below D_2 and D) is a non-dimensional value, it is equivalent to short crack a_1 discussed as reference[1-3], D_{tr} is a transition damage value between two stages from micro to macro damage growth process. Here must put up conversion for dimensions and units, and must be defined in 1mm (1 millimeter) of crack length equivalent to one of damage-unite (1 damage unit), in 1m (1 meter) equivalent to 1000 of damage-unit (1000 damage units). The I'_1 is defined as damage strain factor, that is driving force of damage growth under monotonous load; $\Delta I'_1$ is defined as damage strain factor range, that is driving force of damage growth under fatigue loading, their units are "$\%^{m_1^{'}} \cdot damage - nuit - namber$", in practice it is also a non-dimensional value. ε'_f is a fatigue ductility factor, m'_1 is fatigue ductility exponent, $m'_1 = -1/c_1^{'}, c_1^{'}$ just is also a fatigue ductility exponent under low cycle fatigue. The $B_1^{'}$ is comprehensive material constants, its physical meaning is a concept of power, is a maximal increment value to give out energy for damage growth in one cycle before failure.

Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 2 on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m'_1 of the formula (4). So the $B_1^{'}$ is a calculable comprehensive material constants.

Where

$$v'_{eff} = \frac{\ln(D_{1f} / D_0)}{N_{1fc} - N_{01}} = \frac{[\ln(D_{1f}/D_0) - \ln D_1/D_{01})]}{N_{1f} - N_{01}} \quad (5)$$

$$(damage - unit - number/cycle)$$

or $v'_{eff} = \frac{D_{1f} \ln(1/1 - \psi)}{N_{1f} - N_{01}}$ (6)

$$(damage - unit - number/cycle)$$

The v'_{eff} in eqn (4-6) is defined as an effective rate correction factor in first stage, its physical meaning is the effective damage rate to cause whole failure of specimen material in a cycle, its unit is the $damage - unit - number / cycle$. ψ is a reduction of area. D_0 is pre-micro-damage value which has no effect on fatigue damage under prior cycle loading [9]. D_{01} is an initial damage value, D_f is a critical damage value before failure, N_{01} is initial life in first stage, $N_{01} = 0$; N_{1f} is failure life, $N_{1f} = 1$. Such, its final expansion equation for eqn. (1) is as following form,

$$N_1 = \frac{\ln D_{tr} - \ln D_1}{2(2\varepsilon'_f)^{m'_1} \times (v_{eff})^{-1} (\Delta\varepsilon_p)^{m'_1}}, \quad (7)$$

$$(Cycle), (\sigma > \sigma_s)$$

If materials occur strain hardening, and want to via the stress σ to express it, due to plastic strain occur cyclic hysteresis loop effect, then the life predicting equation corresponded to reversed direction curve C_1C in Fig1 should be

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{A_1^{'} \times (\Delta H'_1 / 2)^{m_1}} (Cycle), (\sigma > \sigma_s) \quad (8)$$

Here

$$H'_1 = \sigma \cdot D_1^{1/m_1} \quad (9)$$

$$\Delta H'_1 = \Delta\sigma \cdot D_1^{1/m_1} \quad (10)$$

H'_1 is defined as the damage stress factor, the $\Delta H'_1 / 2$ is damage stress factor amplitude. Same, that H'_1 is driving force of damage evolving under monotonous loading, and the $\Delta H'_1$ is driving force of under fatigue loading. Its physical and geometrical meanings of the $A_1^{'}$ are similar to the $B_1^{'}$. $A_1^{'}$ is also calculable comprehensive material constant, for $\sigma_m = 0$, it is as below

$$A_1^{'} = 2(2\sigma_f^{'})^{-m_1}(v_{eff})^{-1}, (\sigma_m = 0) \quad (11)$$

But if $\sigma_m \neq 0$, here the correction for mean stress, to adopt a method in reference [10], it is as follow

$$A_1^{'} = 2[2\sigma_f^{'}(1 - \sigma_m/\sigma_f)]^{-m_1}(v_{eff})^{-1}, (\sigma_m \neq 0) \quad (12)$$

Or

$$A'_1 = 2K'^{-m_1}[2\varepsilon'_f(1 - \sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1}$$
$$(\sigma_m \neq 0) \quad (13)$$

Where the $\sigma_f^{'}$ is a fatigue strength coefficient, K' is a cyclic strength coefficient. $m_1 = -1/b'_1$, m_1 and b'_1 are the fatigue strength exponent. $m_1 = -1/c'_1 \times n'$, $n' = b_1^{'}/c_1^{'}$, n' is a strain hardening exponent. So that, the final expansion equation for (8) is as below form,

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2[2\sigma_f^{'}]^{-m_1}(v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1}} \quad (14)$$
$$(\sigma > \sigma_s, \sigma_m = 0)$$

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2[2\sigma_f^{'}(1 - \sigma_m/\sigma_f)]^{-m_1}(v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1}} \quad (15)$$
$$(\sigma > \sigma_s, \sigma_m \neq 0)$$

If take formula (13) to replace the A'_1 in equation (8), its final expansion equation is as below forming

(16)

Here influence of mean stress in eqn (15-16) can be ignored.

2)The two parameter multiplication method

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2K'^{-m_1}[2\varepsilon'_f(1 - \sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1}}$$
$$(\sigma > \sigma_s, \sigma_m \neq 0)$$

Same, under $\sigma > \sigma_s$ condition, if adopt the two parameter multiplication method to express life equation corresponded to reversed direction curve C_1C, it is as following

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{A'^{*}_1 \times (0.25\Delta Q'_1)^{\frac{m_1 m'_1}{m_1 + m'_1}}} (Cycle) \quad (17\text{-}1)$$

or

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{A_1^{*} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}} \times D_1} (Cycle) \quad (17\text{-}2)$$

Where the Q'_1 is defined as the damage Q'_1-factor of two-parameter, the $\Delta Q'_1$ is defined the damage Q'_1-factor range of two-parameter.

$$Q'_1 = (\varepsilon \cdot \sigma)\ D_1^{1/\frac{m_1 m'_1}{m_1 + m'_1}} \quad (18)$$

$$\Delta Q'_1 = (\Delta\varepsilon \cdot \Delta\sigma)\ D_1^{1/\frac{m_1 m'_1}{m_1 + m'_1}} \quad (19)$$

$$A'^{*}_1 = 2[4(\sigma'_f\ \varepsilon'_f)]^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1},$$
,
$$(\mathrm{MPa}^{\frac{m_1 m'_1}{m_1 + m'_1}} \mathrm{damage\text{-}nuit\text{-}number}/cycle)$$
$$(\sigma_m = 0) \quad (20)$$

$$A'^{*}_1 = 2[4(\sigma'_f\ \varepsilon'_f)(1 - \sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1},$$
$$(\mathrm{MPa}^{\frac{m_1 m'_1}{m_1 + m'_1}} \mathrm{damage\text{-}nuit\text{-}number}/cycle)(\sigma_m \neq 0)$$
(21)

Same, the eqn (18) is driving force of micro-damage under monotonic loading, and the eqn (19) is driving force under fatigue loading. It should be point that, the parameter A'^{*}_1 in eqn (17) is also a comprehensive material constant. Its physical and geometrical meaning of the A_1^{*} is similar to above the $A_1^{'}$. And its slope of micro-trapezium bevel edge just is corresponding to the exponent $m_1 m'/(m_1 + m'_1)$ of the formula (20-21). By the way, here is also to adopt those material constants $\sigma'_f, b_1^{'}, \varepsilon'_f, c_1^{'}$ as "genes" in the fatigue damage subject [3]. Therefore, for the eqn (17), its final expansion equation corresponded reversed to curve 2' (C_1C) (Fig 1.) is as below form:

$$N_1 = \frac{\ln D_{tr} - \ln D_1}{2(4\sigma'_f\ \varepsilon'_f)^{-\frac{m_1 m'_1}{m_1 + m'_1}} \times (v_{eff})^{-1} \times (0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1 + m'_1}}} \quad (22)$$
$$, (Cycle), (\sigma_m = 0)$$

$$N_{oi} = \frac{\ln D_{oi} - \ln D_{01}}{2[4(\sigma'_f \varepsilon'_f)(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1}} \times \frac{1}{\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}}}, (\sigma_m \neq 0) \quad (23)$$

Here, influence of mean stress in eqn (23) can also be ignored. But it must point that the total strain range $\Delta\varepsilon$ in eqn (22-23) should be calculated by Masing law as following eqn.[11]

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \quad (24)$$

B) The Calculations for macro-damage process

1) The single parameter method

Under $\sigma > \sigma_s$ condition, due to the material behavior come into the macro-damage stage, the exponent in life equation also to show change from m_1' to λ_2; and due to still occur cyclic hysteresis loop effect, its life models corresponded to reversed curve C_2C_1 in figure 1 is as below form

$$N_{2eff} = \int_{D_{tr}}^{D_{2eff}} \frac{dD_2}{B_2' \times [y_2(a/b)\Delta\delta'_t/2]^{\lambda_2}} (Cycle), (\sigma > \sigma_s) \quad (25)$$

Where

$$\delta'_t = 0.5\pi \times \sigma_s \times D_2(\sigma/\sigma_s + 1)/E, \quad (26)$$

$$\Delta\delta'_t = 0.5\pi \times \sigma_s \times D_2(\Delta\sigma/2\sigma_s + 1)/E, \quad (27)$$

Here δ'_t is a defined as the damage crack tip open displacement, that is equivalent to the crack tip open displacement, it is the driving force under monotonous load. Here must define the “1-damage unit crack tip open displacement” value equivalent to “1 mm crack tip open displacement” value. $\Delta\delta'_t$ is defined as the damage crack tip open displacement range, it is the driving force under repeated loading. The $y_2(a/b)$ is correction factor [12] related to long crack form and structure size. Here should note the B_2' is also defined as a calculable comprehensive material constant,

$$B'_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)D_{2eff}/E)]^{\lambda_2} \times v_{pv}, (\sigma_m = 0) \quad (28\text{-}1)$$

$$B'_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1-\sigma_m/\sigma'_f)D_{2eff}/E)]^{\lambda_2} \times v_{pv}, (\sigma_m \neq 0) \quad (28\text{-}2)$$

$$v'_{pv} = \frac{(D_{2pv} - D_{02})}{N_{pv} - N_{02}} \approx 3\times10^{-5} \sim 3\times10^{-4} = v^* \quad (damage-unit-number/Cycle) \quad (29)$$

Where λ_2 is a ductility exponent in macro damage process, $\lambda_2 = -1/c_2'$, c_2' is a fatigue ductility exponent under low cycle. the v_{pv} is defined to be the virtual damage rate, its physical meaning is an effective damage rate to cause whole failure of specimen material in a cycle in the second stage, its unit is $damage-unit-number/Cycle$, its value is similar to the factor v^*-value in reference [13], but both units are different, where is the “$m/Cycle$”. The D_{2pv} is a virtual damage value, D_{02} is an initial damage value as equivalent to a precrack size. N_{02} is an initial life, $N_{02} = 0$. N_{pv} is a virtual life, $N_{pv} = 1$.

So that, the final expansion equations is derived from above mentioned eqn. (25) as follow

For $\sigma_m = 0$,

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(D_{2eff}^{1-\lambda_2} - D_{02}^{1-\lambda_2})}{2[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)D_{2eff}/E)]^{\lambda_2} \times v_{pv}} \times \frac{1}{\left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s + 1)}{E}\right]^{\lambda_2}}, (cycle) \quad (30)$$

For $\sigma_m \neq 0$, it should be

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(D_{2eff}^{1-\lambda_2} - D_{02}^{1-\lambda_2})}{2[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1-\sigma_m/\sigma'_f)D_{2eff}/E)]^{\lambda_2} \times v_{pv}} \times \frac{1}{\left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)}{E}\right]^{\lambda_2}}, (cycle) \quad (31)$$

Where, influence to mean stress can usually ignored in the eqn (31). D_{2eff} is an effective damage value, it can calculate from effective damage crack tip opening displacement δ'_{2eff}

$$D_{2eff} = \frac{E \times \delta'_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s + 1)}, (damage-unit-number) \quad (32)$$

And

$$\delta'_{2eff} = (0.25 \sim 0.4)\delta'_c, (damage-unit-number) \quad (33)$$

Here the δ'_c is critical damage crack tip displacement, it is equivalent the critical crack tip displacement δ_c in fracture mechanics, both is only on the unit to be different. So the D_{2eff} in (30-31) can be converted and calculated out by δ_c -value in "1mm" value equivalent to "1 damage-unit" by means of equations (32-33). It must be point that the life units in eqns (25,30-31) are all cyclic number.

2) The two parameter multiplication method

With two-parameter-multiplication method to calculate the life in the second stage, it can yet use two kinds of methods: the Q'_2 -factor method and the σ -stress method.

a) Q'_2 -Factor method

To use Q'_2 -factor method calculating the macro damage life, here its effective life models corresponded to reversed curve C_2C_1 in figure 1 is as below form

$$N_{2eff}=\int_{D_{tr}}^{D_{eff}}\frac{dD_2}{B'^*_2\times(0.25y_2(a/b)\Delta Q'_2)^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}}} \quad (34)$$

$$(Cycle),(\sigma>\sigma_s)$$

Where

$$Q'_2=y_2(a/b)K'_1\delta'_t,$$

$$(MPa\cdot\sqrt{damage-unit-number}\cdot damage-unit-number) \quad (35)$$

$$\Delta Q'_2=y_2(a/b)(\Delta K'_2\cdot\Delta\delta'_t),$$

$$(MPa\cdot\sqrt{damage-unit-number}\cdot damage-unit-number) \quad (36)$$

$$K'_1=\sigma\sqrt{\pi D_2},(MPa\sqrt{danage-unit-number} \quad (37)$$

The Q'_2 -factor and $\Delta Q'_2$ are all macro damage driving force, which are respectively under monotonous and repeated loading, their unit are all „ $MPa\cdot\sqrt{damage-unit-number}\cdot damage-unit-number$ ". The K'_1 is defined as damage stress intensity factor, the unit is also " $MPa\cdot\sqrt{damage-unit-number}$, it is equivalent to stress intensity factor; the δ'_t is a damage crack tip open displacement, its unit is dimensionless, by " $damage-unit-number$ " to express. B'^*_2 is also calculable comprehensive material constant, on exponent as compared with above eqn (20) and (21) that is not different.

$$B'^*_2=2[4(K'_{2fc}\,\delta'_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}}\times v'_{pv},(\sigma_m=0)$$

$$,(MPa^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}}\cdot damage-unit-number/cycle \quad (38\text{-}1)$$

$$B'^*_2=2[4K'_{2fc}\,\delta'_{2fc}(1-K'_{2m}/K'_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}}\times v'_{pv},(\sigma_m\neq 0) \quad (38\text{-}2)$$

Where m_2 is an linear elastic exponent in long crack growth process, $m_2=-1/b'_2$. And λ_2 is a ductility exponent, $\lambda_2=-1/c'_2$.

And the effective life expanded equation corresponded to reversed direction curve D_2D_1 should be

For $\sigma_m=0$

$$N_{2eff}=\frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(D_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}-D_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2[4K'_{2c}\,\delta'_{2c}]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v'_{pv}\times[0.25y_2(a/b)\Delta K'_2\cdot\Delta\delta'_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}},(Cycle) \quad (39\text{-}1)$$

For $\sigma_m\neq 0$

$$N_{2eff}=\frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(D_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}-D_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2[4K'_{2fc}\,\delta'_{2fc}(1-K'_{2m}/K'_{2fc})]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v'_{pv}}$$

$$\times\frac{1}{[0.25y_2(a/b)\Delta K'_2\cdot\Delta\delta'_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}},(Cycle) \quad (39\text{-}2)$$

In reference [14-15] refer to the effective stress intensity factor in fracture mechanics. Same, here there are also two effective values K'_{2eff} and δ'_{2eff} corresponding to the critical K'_{2fc} and δ'_{2fc} , to propose as follow,

$$K_{2eff}\approx(0.25-0.4)K'_{2fc}\,;\ \delta'_{2eff}=(0.25-0.4)\delta'_{2fc} \quad (40)$$

Where the D_{2eff} in (39) is an effective damage value, it is obtained and calculated from eqns (32-33), (40) and to take less value.

b) σ -Stress method

If adopt stress to express it, the $\Delta Q'_2$ and B^*_2 in eqn (34) are all to express by the stress σ , it should be as follow

$$Q'_2 = 0.5\, y_2(a/b)\sigma \cdot \sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s + 1)]/E \quad (41)$$

$$\Delta Q'_2 = 0.5\, y_2(a/b)\sigma \cdot \sigma_s(\sqrt{\pi D_2})^3(\Delta\sigma/2\sigma_s + 1)]/E \quad (42)$$

For $\sigma = 0$

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2f}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v'_{pv}, \quad (43)$$

$$(MPa^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \cdot damage-unit-number/cycle)$$

For $\sigma \neq 0$

$$B'^*_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2f}})^3] \times (1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \quad (44)$$

$$(MPa^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \cdot damage-unit-number/cycle)$$

Therefore the residual life equation of corresponded to reversed direction curve D_2D_1 in fig.1, its final expansion equation is as below form,

For $\sigma_m = 0$,

$$N_{2eff} = \frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(D_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}} - D_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}} \times \frac{1}{\left([y_2(a/b)0.5\,\sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}, (cycle) \quad (45)$$

For $\sigma_m \neq 0$,

$$N_{2eff} = \frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(D_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}} - D_{02}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}} \times \frac{1}{\left([y_2(a/b)0.5\,\sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}, (cycle) \quad (46)$$

C. The damage life prediction calculations in whole process

1) The single parameter method

In damage growth process, for availing to life calculation in whole process, it should take a damage value D_{tr} of transition point between two stages from micro to macro damage evolving process, and the transition point D_{tr} can be derived to make equal between the damage rate equations in two stages, for instance,

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = dD_2/dN_2 \quad (47)$$

Here the equation is defined as the damage rate linking equations. For $\sigma_m \neq 0$, if to select driving force equations (10) and (27), and for relative comprehensive material constant A_1' and B_2' to select formula (13) and (29), so its expanded damage rate linking equation for eqn (47) corresponding to positive curve CC_1C_2 is as following form,

$$\frac{dD_1}{dN} = \left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (v_f \times D_{tr})^{-1} \times (\Delta\sigma/2)^{m_1} \times D\right\}_{D_{01}\to D_{tr}}$$
$$= \frac{dD_{tr}}{dN} = \frac{dD_{2tr}}{dN_2} = \left\{\begin{array}{l} 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{eff}/E)\right]^{\lambda_2} \times v_{pv} \\ \times\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D}{E}\right]^{\lambda_2}\end{array}\right\}_{D_{tr}\to D_{eff}},$$

$$damage-unit-number/cycle, (\sigma \neq 0) \quad (48)$$

Moreover the life equations in whole process corresponding to reversed direction curve C_2C_1C should be as following

$$\sum N = N_1 + N_2 = \int_{D_{01}}^{D_{tr}} \frac{dD}{A'_1 \times (\Delta\sigma/2)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{B'_2(\Delta\delta'_t/2)^{\lambda_2}} \quad (49)$$

The life prediction expanded expression in whole process corresponded reversed curve C_2C_1C, it should be

$$\sum N = \int_{D_{01}}^{D_{tr}} \frac{dD}{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{1}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{2eff}/E)\right]^{\lambda_2} \times v_{pv}} \times \frac{dD}{\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D}{E}\right]^{\lambda_2}}, (cycle) \quad (50)$$

2) The two parameter multiplication method

For two parameter multiplication method, if in $\sigma_m \neq 0$ as example, its expanded damage rate linking equation for (47) corresponding to positive curve CC_1C_2 is as following form

$$\frac{dD_{1tr}}{dN_1}=\left\{2\cdot[4\sigma'_f\,\varepsilon'_f\,(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1}\times(0.25\times\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}\right\}_{D_{01}->D_{tr}}$$

$$=\frac{dD_{tr}}{dN}=\frac{dD_{2tr}}{dN_2}=$$

$$=\left\{\begin{array}{l}2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times\\ \times\left([0.5\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\end{array}\right\}_{D_{tr}->D_{eff}},$$

$$damage-unit-number/cycle,.(\sigma\neq 0)$$

(51)

The life equations in whole process corresponding to reversed direction curve C_2C_1C should be as following

$$\sum N=N_1+N_2=\int_{D_{01}}^{D_{tr}}\frac{dD}{A_1^*\times(\Delta Q_1/2)^{m_1}\times D}+\int_{D_{tr}}^{D_{2eff}}\frac{dD}{B_2^*(\Delta Q_2/2)^{\lambda_2}}$$

(52)

And the life prediction expanded expression in whole process, corresponding reversed curve C_2C_1C, it should be

$$\sum N=\int_{D_{01}}^{D_{tr}}\frac{dD}{2[4\sigma'_f\,\varepsilon'_f\,(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1}}$$

$$\times\frac{1}{\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}}+$$

$$+\int_{a_{tr}}^{D_{2eff}}\frac{dD}{2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}}$$

$$\times\frac{1}{v_{pv}}\times\frac{1}{\left([0.5\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}$$

(53)

It should yet point that the calculations for rate and life in whole process should be according to different condition, to select appropriate calculable equation. And have to explain that its meaning of the damage rate linking equations (48,51) is to make a link relation between the first stage rate and the second stage rate, which it should be calculated by the micro damage growth rate equation before the damage value D_{tr} at transition point; it should be calculated by the macro damage growth rate equation after the damage value D_{tr}, that is not been added together by the rates for two stages. But the life calculations for two stages can be added together. About calculation method, it can calculate by means of computer doing computing by different crack size [16,17].

III. CALCULATING EXAMPLE

A. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b=573MPa$, yield limit $\sigma_s=361MPa$, fatigue limit $\sigma_{-1}=267.2MPa$, reduction of area is $\psi=0.51$, modulus of elasticity $E=200000MPa$; Cyclic strength coefficient $K'=1165MPa$, strain-hardening exponent $n'=0.187$; Fatigue strength coefficient $\sigma'_f=947.1MPa$, fatigue strength exponent $b'_1=-0.111$, $m_1=9.009$; Fatigue ductility coefficient $\varepsilon'_f=0.464$, fatigue ductility exponent $c'_1=-0.5395$, $m'_1=1.8536$. Threshold value $\Delta K_{th}=8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c}=K_{1c}=92.7\text{MPa}\sqrt{\text{m}}$, critical damage stress intensity factor $K'_{2c}=92.7MPa\sqrt{1000-damage-unit}$ of equivalent to the $K_{1c}(K_{2c})$. Its working stress $\sigma_{max}=450MPa$, $\sigma_{min}=0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b)=1$. Other computing data are all included in table 1-2.

TABLE I. COMPUTING DATA

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv}
92.7	28.23	8.6	2×10^{-4}

TABLE II. COMPUTING DATA

m_2	δ_c, mm	λ_2	$y_2(a/b)$	a'_{th}, mm
3.91	0.18	2.9	1.0	0.07

B. Required Calculating Data

Try by two kinds of calculating methods to calculate respectively as following different data and depicting their curves :

- To calculate the transitional point damage value D_{tr} between two stages;

- To calculate the damage rate at transitional point (at damage value D_{tr});

- To calculate the life N_1 in first stage from micro damage value $D_1 = 0.02 damage-unit$ growth to transitional point D_{tr} ;

- To calculate the life N_2 in second stage N_2 from transitional point D_{tr} to macro damage value $D_{2eff} = 5-damage-unit$;

- Calculating the whole service lifetime $\sum N$;

- Depicting their damage life curves in whole process.

C. Calculations for Relevant Parameters

1) Donversions for dimensions and units

Data after conversions for dimensions and units is including in table 3, 4, and 5.

TABLE III. COMPUTING DATA AFTER CONVERSIONS

$K'_{1c}, MPa\sqrt{1000damage-nuit}$	$K'_{eff}, MPa\sqrt{1000damage-nuit}$
92.7	28.23

TABLE IV. COMPUTING DATA AFTER CONVERSIONS

$K'_{th}, MPaMPa\sqrt{1000damage-nuit}$	v'_{pv}	m_2
8.6	2×10^{-4}	3.91

TABLE V. COMPUTING DATA AFTER CONVERSIONS

$\delta'_c\ damage-unit$	λ_2	$y_2(a/b)$	$D_{eff} damage-unit$
0.18	2.9	1.0	2

2) Calculations for relevant parameters

- Stress range calculation:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450(MPa)$$

- Mean stress calculation: $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (450-0)/2 = 225MPa$

- *According to formulas (6), calculation for correction coefficient v'_{eff} in first stage*

$$v'_{eff} = D_{eff}\ln[1/(1-\psi)] = 2\times\ln[1/(1-0.51)] = 1.43,$$
(damage - unit/cycle)

- *By eqn (29), to select virtual rate v'_{pv} in second stage, here take:*

$$v_{pv} = \frac{D_{2eff} - D_{02}}{N_{2f} - N_{02}} \approx 2.0\times10^{-4}(damage-unit/Cycle),$$

$N_{2f} = 1,\ \ N_{02} = 0$

- *According to formulas (32), Calculating effective size a_{eff}*

$$D_{eff} = \frac{E\times\delta'_{eff}}{\pi\sigma_s(\sigma_f/\sigma_s + 1)} = \frac{200000\times0.25\times0.18}{\pi361(947.1/361+1)}$$
$$= 2.1(damage-unit),$$

Take $D_{eff} = 2.0mm$.

Here take effective damage value in first and the second stage:

$$D_{1eff} = D_{2eff} = 2damage-unit$$

Beneath is also to calculate by means of two kinds of methods respectively.

D. The Concrete Calculation Methods and Processes

1) The single parameter method

The concrete calculation methods and processes are

as follows:

a)To calculate the transitional point damage value D_{tr} between two stages

By the rate link formulas (47-48), select relevant equation for micro damage rate calculation.

- At first, calculation for comprehensive material constant A'_1 by eqn (13)

$$A'_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'}\times(D_{ef}\times v_f)^{-1}$$
$$= 2\times1165^{-9.01}\times[2\times0.464(1-225/947.1)]^{1/-0.5395}$$
$$\times(2\times0.713)^{-1} = 6.28\times10^{-28},(MPa^{m_1}\sqrt{damage-unit})^{-m_1}$$

- Select the damage rate linking equation (48), and for damage rate in first stage to take brief calculations as follow form,

$$dD_1/dN_1 = A'_1\times(\Delta\sigma/2)^{m_1}\times D_1 = 3.193\times10^{-28}$$
$$\times(450/2)^{9.01}\times D_1 = 6.28\times10^{-28}\times1.56\times10^{21}\times D_1$$
$$= 9.8\times10^{-7}\times D_1$$

Still by the rate link formulas (48), calculating for macro damage rate in second stage

- Calculation for comprehensive material constant B_2 by eqn (28)

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{eff}/E)\right]^{\lambda_2}\times v_{pv}$$
$$= 2\left[2(3.1416\times361(947.1/361+1)(1-225/947.1)\times2/200000)\right]^{2.9}$$
$$\times2\times10^{-4} = 9.1988,(damage-unit)^{-\lambda_2}\times damage-unit/Cycle$$

For the damage rate in second stage, to take brief calculations as follow form,

$$dD_2/dN_2 = B_2\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2}$$
$$=9.1988\times\left[\frac{0.5\pi 361(450/(2\times 361)+1)D_2}{E}\right]^{2.9}$$
$$=9.1988\times 1.6698\times 10^{-7}D_2^{2.9}$$
$$=1.5384\times 10^{-6}D_2^{2.9}$$

- Calculation for transitional point damage value D_{tr}

According to the equations (47) and (48), for the transitional point damage value D_{tr} between two stages to do calculation; then, to take brief link calculation formulas as follow form,

$6.28\times 10^{-28}\times 1.56\times 10^{21}\times D_{tr} = 9.1988\times 1.6698\times 10^{-7}\times D_{tr}^{2.9}$,

$D_{tr} = (0.638)^{\frac{1}{1.9}} = (0.638)^{0.5263} = 0.789(damage-unit)$.

So to obtain the transitional point damage value $D_{tr} = 0.789(damage-unit)$ between two stages.

b) To calculate the damage rate at transitional point (damage value D_{tr})

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = 9.8\times 10^{-7}D_1$$
$$=9.8\times 10^{-7}\times 0.789 = 7.74\times 10^{-7}(damage-unit/cycle)$$

$$dD_2/dN_2 = dD_{tr}/dN_{tr} = 1.5384\times 10^{-6}D_{tr}^{2.9}$$
$$=1.5384\times 10^{-6}\times(0.79)^{2.9}$$
$$=7.74\times 10^{-7}(damage-unit/cycle)$$

Here can be seen, the damage rate at the transition point $(D_{tr} = 0.789damage-unit)$ is same.

And above damage rate value equivalent to the crack growth rate at the transition point of crack size $a_{tr} = 0.789(mm)$, it is $da_{tr}/dN_{tr} = 7.74\times 10^{-7}(mm/cycle)$.

c) Life prediction calculations in whole process

- Predicting life in first stage N_1

To select eqn (16), the life N_1 from micro-damage $D_1 = 0.02damage-unit$ to transitional point (damage value $D_{tr} = 0.789damage-unit$) is as follow,

$$N_1 = \frac{\ln D_{tr} - \ln D_{01}}{2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'}\times(D_{eff}\times v_f)^{-1}}$$
$$\times\frac{1}{(\Delta\sigma/2)^{m_1}\times D} =$$
$$=\frac{\ln 0.789 - \ln 0.02}{2\times 1165^{-9.01}\times[2\times 0.464(1-225/947.1)]^{1/-0.5395}(2\times 0.713)^{-1}}$$
$$\times\frac{1}{(450/2)^{9.01}} = \frac{3.675}{6.28\times 10^{-28}\times 1.56\times 10^{21}} = \frac{3.675}{9.8\times 10^{-7}}$$
$$=3751260(Cycle)$$

So predicting life in first stage $N_1 = 3751260(Cycle)$

And for above formulas, we can derive simplified life equation in first stage corresponded to different damage value as follow form

$$N_1 = \frac{1}{9.8\times 10^{-7}D_1}$$

- Predicting life in second stage N_2

To select eqn (31), to calculate the life N_2 in second stage from transitional point damage value $D_{tr} = 0.789(damage-unit)$ to $D_{2eff} = 5damage-unit$ is as follow,

$$N_2 = \frac{\frac{1}{1-\lambda_2}(D_{2eff}^{\ 1-\lambda_2} - D_{tr}^{\ 1-\lambda_2})}{2[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{eff}/E)]^{\lambda_2}\times v_{pv}}$$
$$\times\frac{1}{\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)}{E}\right]^{\lambda_2}} =$$
$$=\frac{\frac{1}{1-2.9}(5^{1-2.9} - 0.789^{1-2.9})}{2[(3.1416\times 361(947.1/361+1)(1-225/947.1)\times 2/200000)]^{2.9}}$$
$$\times\frac{1}{v_{pv}}\times\frac{1}{\left[\frac{0.5\pi 361(450/(2\times 361)+1)}{E}\right]^{2.9}} = \frac{0.8}{9.1988\times 1.6698\times 10^{-7}}$$
$$=\frac{0.8}{1.5384\times 10^{-6}} = 520625(Cycle)$$

From above formulas, we can also derive simplified life equation corresponding different damage value as follow form

$$\rightarrow N_2 = \frac{1}{1.5384\times 10^{-6}D_2^{2.9}}$$

- Life prediction calculations in whole process

Therefore, predicting life in whole process is

$$\sum N = N_1 + N_2 = 3751260 + 520625 = 4271885(Cycle)$$

The life data corresponded to different damage values which are all converted to the relation between the crack growth sizes and the life, and are all included in tables 6~13.

2) The two parameter multiplication method

a) To calculate the transitional point damage value D_{tr} between two stages

- Calculation for comprehensive material constant $A_1'^*$ in first stage by eqn (21)

$$A_1'^* = 2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (D_{eff} \times v_f)^{-1}$$

$$= 2[4(947.1\times 0.464)(1-225/947.1)]^{-\frac{9.009\times 1.8536}{9.009+1.8536}}$$

$$\times (2\times 0.713)^{-1} = 2.216\times 10^{-5} (\text{MPa}^{\frac{m_1 m'_1}{m_1+m'_1}} \text{damage-nuit-number}/cycle)$$

- Calculation for comprehensive material constant B_2^* in second stage by eqn (44)

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}$$

$$\times v_{pv} = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{\frac{m_2\lambda_2}{m_2+\lambda_2}}$$

$$= 2\{[\frac{947.1\times 361(947.1/361+1)}{200000}(\sqrt{\pi\times a_{2eff}})^3](1-225/947.1)\}^{-\frac{3.91\times 2.9}{3.91+2.9}}$$

$$\times 2\times 10^{-4} = 2\{[6.1945\times \pi^{1.5} 2^{1.5}]0.7624\}^{-1.665} \times 2\times 10^{-4}$$

$$= 2\{74.381\}^{-1.665} \times 2\times 10^{-4} = 3.0625\times 10^{-7}$$

$$(MPa^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \cdot damage-unit-number/cycle)$$

- According to the equations (47) and (51), calculation for damage value D_{tr} at transitional point.

Then, for the transitional point value D_{tr} between two stages doing calculation, same, it can make equal between the rate expansion equation at left side and at right side as following.

$$dD_1/dN_1 = dD_{tr}/dN_{1-2} = dD_2/dN_2$$

$$A_1'^* \times (0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}} \times D_{tr}$$

$$= A_2'^*[(\Delta\sigma/2)\cdot 0.5\sigma_s(\sqrt{\pi D_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}$$

$$2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (D_{1eff} \times v_f)^{-1}$$

$$\times (0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}} \times D_{tr}$$

$$= 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}$$

$$\times [0.5(\Delta\sigma/2)\cdot\sigma_s(\sqrt{\pi D_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}$$

$$2[4(947.1\times 0.464)(1-225/947.1)]^{-\frac{9.009\times 1.8536}{9.009+1.8536}}$$

$$\times (2\ \times 0.7133)^{-1} \times (0.25\times 2.553\times 10^{-3}\times 450)^{\frac{9.009\times 1.8536}{9.009+1.8536}} \times D_{tr}$$

$$= \{[\frac{947.1\times 361(947.1/361+1)}{200000}(\sqrt{\pi\times 2})^3](1-225/947.1)\}^{-\frac{3.91\times 2.9}{3.91+2.9}}$$

$$\times 2\times 10^{-4} \times [0.5(450/2)\cdot 361(\sqrt{\pi D_{tr}})^3(450/2\times 361+1)/E]^{\frac{3.91\times 2.9}{3.91+2.9}}$$

Here to make simplified calculation:

$$3.22\times 10^{-6} D_{tr} = 2.6695\times 10^{-6} \times D_{tr}^{2.4975},$$

$$D_{tr} = 1.2062^{0.6678} = 1.133(damage-unit).$$

So obtain the transitional point damage value $D_{tr} = 1.133(damage-unit)$ between two stages, that is equivalent to crack size $a_{tr} = 1.113mm$ at this point.

b)The damage rate calculations for transitional point D_{tr}

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = 3.22\times 10^{-6}\times D_{tr} =$$
$$= 3.22\times 10^{-6}\times 1.133 = 3.648\times 10^{-6}$$
$$(damage-unit/cycle)$$

$$dD_2/dN_2 = dD_{tr}/dN_{tr} = 2.6695\times 10^{-6} a_{tr}^{2.4975}$$
$$= 2.6695\times 10^{-6}\times 1.133^{2.4975} = 3.646\times 10^{-6}(damage-unit/cycle)$$

That is equivalent to crack growth rate 3.648×10^{-6} $(mm/cycle)$

Thus it can be seen, the crack growth rate at the transition point crack size $a_{tr} = 1.113(mm)$ is same, it is 3.646×10^{-6} $(mm/cycle)$.

c) Life prediction calculations in whole process

- Life prediction calculations in first stage：

Select life predicting calculation equation (23), the calculable life N_1 in first stage from micro-damage $D_{01} = 0.02 damage-unit$ to transitional point $D_{tr} = 1.113(damage-unit)$ is as follow,

$$N_1 = \frac{\ln D_{tr} - \ln D_{01}}{2[4\sigma'_f \varepsilon'_f (1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (a_{1eff}\times v_f)^{-1}}$$

$$\times \frac{1}{(0.25\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}}}$$

$$= \frac{\ln 1.133 - \ln 0.02}{2[4(947.1\times 0.464)(1-225/947.1)]^{-\frac{9.009\times 1.8536}{9.009+1.8536}} \times (2\times 0.7133)^{-1}}$$

$$\times \frac{\ln 1.133 - \ln 0.02}{3.22\times 10^{-6}} = \frac{4.0369}{3.22\times 10^{-6}} = 1253693(Cycle)$$

So first stage life $N_1 = 1253693(Cycle)$

And from above we can derive simplified life equation in first stage corresponded to different damage value as follow form

$$N_1 = \frac{1}{3.22\times 10^{-6} D_1}$$

- Life prediction calculations in second stage：

Select life predicting calculation equation (46), the calculable life N_2 in second stage from transitional point damage value $D_{tr} = 1.113(damage-unit)$ to $D_{eff} = 5damage-unit$

is as follow,

$$N_2 = \frac{\frac{4(m_2+\lambda_2)}{4m_2+4\lambda_2-6m_2\lambda_2}\left(D_{2eff}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}} - D_{tr}^{\frac{2m_2+2\lambda_2-3m_2\lambda_2}{2(m_2+\lambda_2)}}\right)}{2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}}$$

$$\times\frac{1}{[y(a/b)0.5(\Delta\sigma/2)\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}}$$

$$= \frac{\frac{4(3.91+2.9)}{4\times 3.91+4\times 2.9-6\times 3.91\times 2.9}}{2\{[\frac{947.1\times 361(947.1/361+1)}{200000}(\sqrt{\pi\times 2})^3](1-225/947.1)\}^{-\frac{3.91\times 2.9}{3.91+2.9}}}$$

$$\times\frac{\left(5^{\frac{2\times 3.91+2\times 2.9-3\times 3.91\times 2.9}{2(3.91+2.9)}} - 1.133^{\frac{2\times 3.91+2\times 2.9-3\times 3.91\times 2.9}{2(3.91+2.9)}}\right)}{2\times 10^{-4}}$$

$$\times\frac{1}{[0.5(450/2)\times 361(\sqrt{\pi a_2})^3(450/2\times 361+1)/200000]^{\frac{3.91\times 2.9}{3.91+2.9}}}$$

$$= \frac{-0.667745256\cdot(5^{-1.4975}-1.133^{-1.4975})}{2.6695\times 10^{-6}} = \frac{0.4939}{2.6695\times 10^{-6}}$$

$$= 185014(cycle)$$

From above we can also derive simplified life equation corresponding different damage value as follow form

$$\rightarrow N_2 = \frac{1}{2.6695\times 10^{-6} D^{2.9}}$$

- Calculation In whole process life

Therefore, whole process life is

$$\sum N = N_1 + N_2 = 1253693 + 185014$$
$$= 1438707(Cycle)$$

The life data corresponded to different damage value in damage growth propagation course is all included in tables 6~13 which are comparisons for life data calculating results in two stages by two kinds of methods.

E. Calculating Results

TABLE VI. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	1	2
Crack size (mm)	0.02	0.04
Single parameter method N1	51020408	25510204
Two parameter method N1	15527950	7763975
Ratio	3.25:1	3.25:1
Single parameter method N2	Invalid section	
Two parameter method N2	Invalid section	

TABLE VII. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	3	4
Crack size (mm)	0.1	0.2
Single parameter method N1	10204082	5102041
Two parameter method N1	3105590	1552795
Ratio	3.25:1	3.25:1
Single parameter method N2		
Two parameter method N2		20856799

TABLE VIII. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	5	
Crack size (mm)	0.4	0.5
Single parameter method N1	2551020	2040816
Two parameter method N1	776398	621118
Ratio	3.25:1	3.25:1
Single parameter method N2		4851966
Two parameter method N2	3693391	2115400
Ratio		2.29:1

TABLE IX. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	6	7
Crack size (mm)	0.6	0.7
Single parameter method N1	1700680	2868041
Two parameter method N1	517598	443656
Ratio	3.25:1	3.25:1
Single parameter method N2	2859513	1828717
Two parameter method N2	1341644	912931
Ratio		2:1

TABLE X. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	Transition point	Transition point
Crack size (mm)	0.789	1.133
Single parameter method N1	1293293	900625
Two parameter method N1	393611	274103
Ratio	3.25:1	3.25:1
Single parameter method N2	1292431	452547
Two parameter method N2	677049	274240
Ratio	1.91:1	1.65:1

TABLE XI. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	9	10
Crack size (mm)	1.5	2.0
Single parameter method N1	680272	510204
Two parameter method N1	207039	155280
Ratio	3.25:1	3.25:1
Single parameter method N2	200570	87085
Two parameter method N2	136076	66336
Ratio	1.47:1	1.31:1

TABLE XII. COMPARISONS FOR LIFE DATA IN TWO STAGES BY TWO KINDS OF METHODS

Data point of number	11	12	13
Crack size (mm)	3.0	4.0	5.0
Single parameter method N1			
Two parameter method N1			
Ratio			
Single parameter method N2	26871	11667	6108
Two parameter method N2	24097	11747	6728
Ratio	1.12:1	0.993:1	0.91:1

From the tables, it is observed that the crack size from 0.02 mm to 2mm （as the first stage）, comparison for their result data calculated by the single parameter method and two parameter method, both ratio is all 3.25:1; And for the crack size from 0.5 mm to 5mm （as the second stage）, comparison for their result data calculated by two kinds of method, both ratio is gradually to reduce from 2.3:1 to 0.91:1.

So, the comparisons to their advantages and disadvantages for two methods them-self, that are: for the single parameter method, its calculation model is simpler; for the two-parameter method, its calculation precision is higher and more moderate in whole process, but its calculation models are more complex. Looked from the overall trend, both calculated result data is basically closer in whole process; especially the life data in second stage is closer.

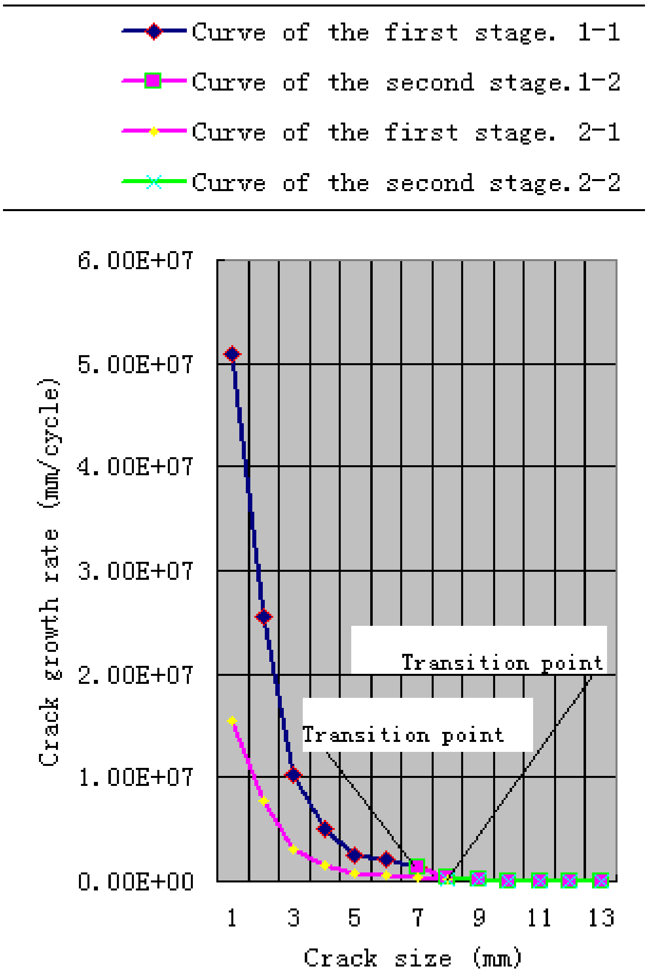

Figure 2. Comparison of life curves in whole process (in decimal coordinate system)

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;

(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;

(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size $0.789mm$).

(D) 2-1---Curve in first stage depicted by two-parameter calculating data;

(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;

(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size $1.113mm$).

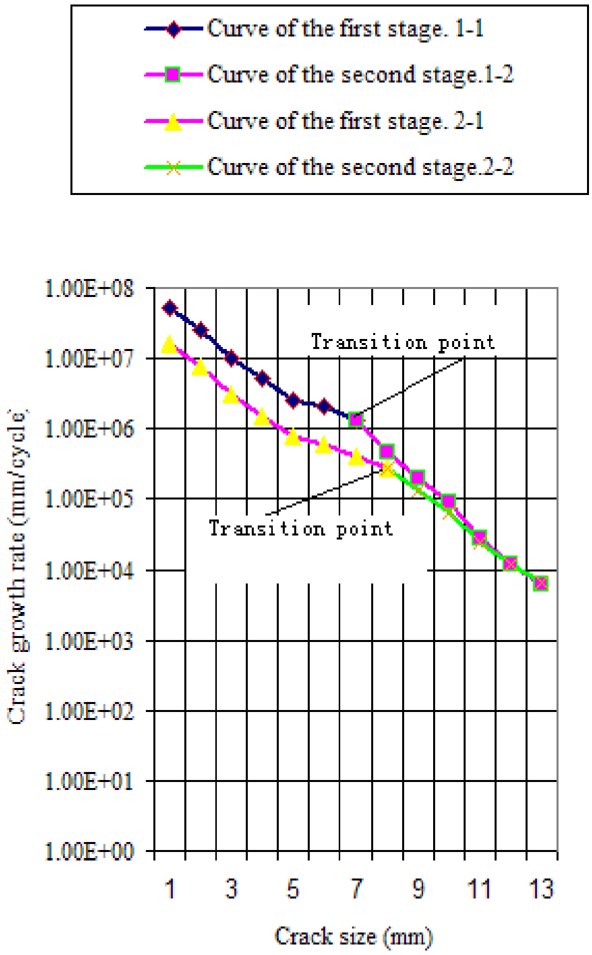

Figure 3. Comparison of life curves in whole process (in logarithmic coordinate system)

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;

(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;

(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size $0.789mm$).

(D) 2-1---Curve in first stage depicted by two-parameter calculating data;

(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;

(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size $1.113mm$).

IV. Conclusions

1) About comparison for calculating methods of two kinds: For the single parameter method, its calculation model is simpler; for the two-parameter method, its calculation precision in whole process is higher and more moderate, but its calculation models are more complex. Looked from the overall trend, the result data calculated by two methods is basically closer in whole process; especially both life data in second stage is closer.

2) About conversion regulations on variables, dimensions and units: Inside mathematical models to convert crack variable a into damage variable D, it must define "1mm-crack-length" equivalent to 'one-damage-unit', "1m-crack-length" equivalent to '1000-damage-unit', this is a key for linking and communicating the damage mechanics and the fracture mechanics.

3) About the methods for whole process rate and life calculations: For damage transition value D_{tr} can be calculated to make equal by between the micro-damage rate and the macro-damage rate equation; For rate calculation, before the transition point D_{tr} it should be calculated by the micro damage rate equation, after the transition point D_{tr} it should be calculated by the macro damage rate equation. But the lifetime calculations can be added together by life cycle number in two stages.

4) About difference cognition for material constants: True material constants must show the inherent characters of materials, such as the σ_s and E, δ, ψ etc in the material mechanics; and for instance the σ_f and σ'_f ; ε_f and ε'_f ; b_1 and b'_1 ; c_1 and c'_1 and so on in the fatigue damage mechanics; which could all be checked and obtained from general handbooks; But some new material constants about the strength, the rate and the life equations in the fracture and damage mechanics can be calculated by means of the relational expressions, e.g. formulas (11-13), (20-21), (24-26), (28), (37-38), etc. Of course, for which have to combine experiments to verify. Therefore for this kind of material constants can be defined as comprehensive materials constants.

5) Total conclusion: Based on the conventional material mechanics is a calculable subject, in consideration of the conventional parameters there are "the hereditary characters", In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions (1)~(4), then make the fatigue and the damage mechanics disciplines become calculable subjects, that will be to exist the possibility.

Acknowledgments 1

At first author sincerely thanks scientists David Broek, Miner, P. C. Paris, Coffin, Manson, Basquin, Y. Murakami, S. Ya. Yaliema, Morrow J D, etc., they have be included or no included in this paper reference, for they have all made out valuable contributions for the fatigue-damage-fracture subjects. Due to they hard research, make to discover the fatigue damage and crack behavioral law for materials, to form the modern fatigue-damage-fracture

mechanics; due to they work like a horse, make to develop the fatigue-damage-fracture mechanics subjects, gain huge benefits for accident analysis, safety design and operation for which are mechanical equipments in engineering fields. Particularly should explain that author cannot have so many of discovery and provide above the calculable mathematical models and the combined figure 1, if have no their research results.

Acknowledgments 2

Author thanks sincerity the Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering gives to support and provides research funds.

REFERENCES

[1] Yu Yangui, Sun Yiming, MaYanghui and XuFeng. "The Computing of intersecting relations for its Strength Problem on Damage and Fracture to Materials with short and long crack". In: International Scholarly Research Network ISRN. Mechanical Engineering Volume, Article ID 876396. http://www.hindawi.com/isrn/me/.(2011).

[2] Yangui Yu. "The Calculations of Evolving Rates Realized with Two of Type Variables in Whole Process for Elastic-Plastic Materials Behaviors under Unsymmetrical Cycle". Mechanical Engineering Research. Canadian Center of Science and Education 2. (2):77-87; ISSN 1927-0607(print) E-ISSN 1927-0615 (Online), 2012.

[3] Yangui Yu. "The Life Predicting Calculations in Whole Process Realized from Micro to Macro Damage with Conventional Materials Constants". American Journal of Science and Technology.Vol. 1, No. 5, pp. 310-328. 2014.

[4] Yu Yangui, Xu Feng, "Studies and Application of Calculation Methods on Small Crack Growth Behaviors for Elastic-plastic Materials", Chinese Journal of Mechanical Engineering, 43, (12), 240-245, (2007), (in Chinese).

[5] YU Yangui, LIU Xiang, ZHANG Chang sheng and TAN Yanhua. "Fatigue damage calculated by Ratio-Method Metallic Material with small crack under un-symmetric Cyclic Loading, Chinese Journal of Mechanical Engineering", 19, (2), 312-316, (2006).

[6] YU Yangui. "Fatigue Damage Calculated by the Ratio-Method to Materials and Its Machine Parts", Chinese Journal of Aeronautics, 16, (3) 157-161, (2003).

[7] Yu Yangui and LIU Xiang. "Studies and Applications of three Kinds of Calculation Methods by Describing Damage Evolving Behaviors for Elastic-Plastic Materials, Chinese Journal of Aeronautics", 19, (1), 52-58,(2006).

[8] Yangui Yu, "Several kinds of Calculation Methods on the Crack growth Rates for Elastic-Plastic Steels". [In: 13th International Conference on fracture (ICF13), (Beijing, 2013) In CD, ID S17-045].

[9] Y. Murakami, S. Sarada, T. Endo. H. Tani-ishi, "Correlations among Growth Law of Small Crack, Low-Cycle Fatigue Law and ,Applicability of Miner's Rule", Engineering Fracture Mechanics, 18, (5) 909-924, (1983).

[10] Morrow, j. D. Fatigue Design handbook, Section 3.2, SAE Advances in Engineering, Society for Automotive Engineers, (Warrendale, PA, 1968), Vol. 4, pp. 21-29.

[11] Masing, G. Eigerspannungen and Verfestigung bein Messing, in: Proceeding of the 2nd International Congress of Applied Mechanics, (Zurich, 1976), pp. 332-335.

[12] S. V. Doronin, et al., Ed. RAN U. E. Soken, Models on the fracture and the strength on technology systems for carry structures, (Novosirsk Science, 2005) , PP. 160-165. (in Russian)

[13] S. Ya. Yaliema, "Correction about Paris's equation and cyclic intensity character of crack", Strength Problem.147, (9) 20-28(1981). (in Russian)

[14] Xian-Kui Zhu, James A. Joyce, Review of fracture toughness (G, K, J, CTOD, CTOA) testing and standardization, Engineering Fracture Mechanics, 85, 1-46, (2012).

[15] U. Zerbst, S. Beretta, G. Kohler, A. Lawton, M. Vormwald, H.Th. Beier, C. Klinger,I. C erny´, J. Rudlin, T. Heckel a, D. Klingbeil, Safe life and damage tolerance aspects of railway axles – A review. Engineering Fracture Mechanics. 98, 214–271 (2013).

[16] Yu Yangui, MaYanghuia, "The Calculation in whole Process Rate Realized with Two of Type Variable under Symmetrical Cycle for Elastic-Plastic Materials Behavior",[in: 19th European Conference on Fracture, (Kazan, Russia, 26-31 August, 2012), In CD, ID 510].

[17] Yu. Yangui, Bi Baoxiang, MaYanghau, Xu Feng. "Damage Calculations in Whole Evolving Process Actualized for the Materials Behaviors of Structure with Cracks to Use Software Technique". In: 12th International Conference on Fracture Proceeding. Ottawa, Canada. 2009; 12-19. CD.

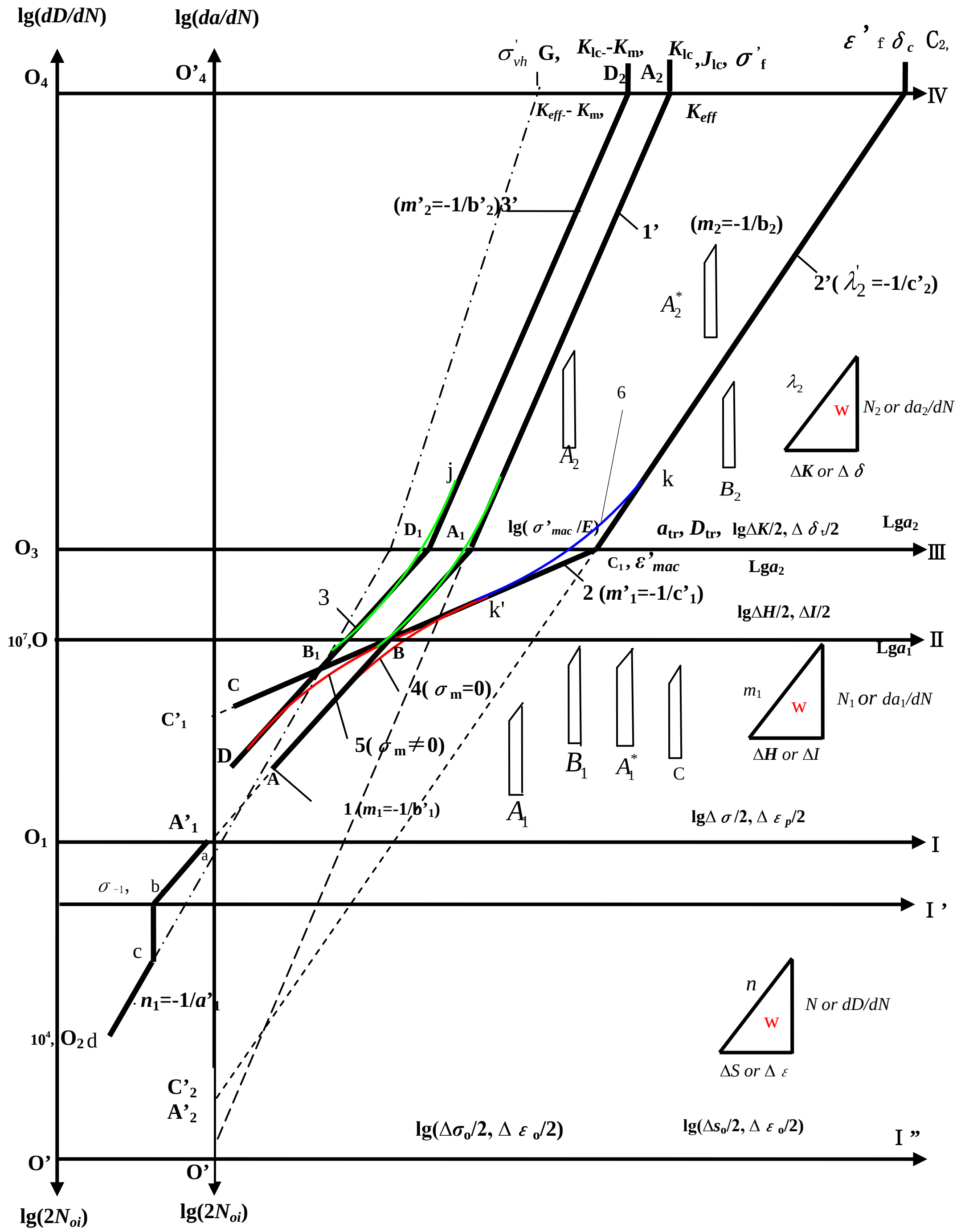

Figure1. Comprehensive figure of materials fatigue-damage-fracture