## IMPACT OF LONGITUDINAL AND TRANSVERSE WAVES BY CYLINDRICAL LAYERS WERE LIQUID

I.I.Safarov, Z.I Boltaev, M.Sh Akhmedov

Bukhara Technological- Institute of Engineering

Republic of Uzbekistan, st. K. Murtazayev 15

safarov54@mail.ru; (+998 93) 625-08-15

We study the field of dynamic stresses and displacements arising near the cylindrical layer (liquid) in a viscoelastic medium during the passage of a plane wave. It is shown that the inclusion of the viscous properties of the material environment in the calculation of the action of harmonic waves, reduces stress and displacement on 10-16%.

**Keywords:** displacement potentials; stressstrain with distance; longitudinal and transverse waves; harmonic waves; Gauss method.

Introduction. Impact of longitudinal and transverse waves on a cylindrical body was investigated by many authors [1, 2, 3, 4]. While considered axially symmetric (and not axially symmetric) tasks used different models for fluid and layers (or shells). Previous works cylindrical body considered as a cylindrical shell and the equation of motion is obtained based on the hypothesis Kirchhoff - Love [5, 6, 7, 8]. Also environment has been viewed as an elastic, t. e. Relationship of stress and strain state obeys Hooke's law [9,10]. This work differs from previous ones in that the cylindrical shell ambient possessing viscous properties, i.e. stress and strain due obeys Boltzmann integral relation -Voltaire [12]. Impact model-tins of longitudinal and transverse waves on cylindrical layers and liquid based on the techniques developed in the dynamics of bodies interacting with a deformable medium, for example, in [11].

Statement of the problem. An infinitely long, homogeneous, isotropic-deformable cylinder with an ideal compressible liquid in an infinite viscoelastic medium, falls harmonic plane wave expansion (or shift) (Fig. 1). The wave front is parallel to the cylinder axis. Thus, the problem of plane strain. Here r = R and outer  $r = R_0$  -inner radii of the cylindrical layer. The main aim of the work is to determine the stress - strain state of cylindrical layer and the environment under the influence of the longitudinal (or transverse) harmonic waves. Under the assumption of generalized plane strain state of the equation of motion in terms of displacements is given by [1]

$$(\tilde{\lambda}_{j}+2\tilde{\mu}_{j})grad \, div \overset{\mathbf{f}}{u}_{j}-\tilde{\mu}rotrot \overset{\mathbf{f}}{u}_{j}+\overset{\mathbf{f}}{b}_{j}=\rho_{j}\frac{\partial^{2}\overset{\mathbf{f}}{u}_{j}}{\partial t^{2}},$$
(1)

where  $\lambda_j$  and  $\mu_j$  (j = 1,2, j = 1- relate to the environment, j = 2- to layer) operator modules of elasticity

$$\begin{split} \widetilde{\lambda}_{j}f(t) &= \lambda_{0j} \Bigg[ f(t) - \int_{-\infty}^{t} R_{\lambda}^{(i)}(t-\tau) f(\tau) d\tau \Bigg] \\ \widetilde{\mu}_{j}f(t) &= \mu_{0j} \Bigg[ f(t) - \int_{-\infty}^{t} R_{\mu}^{(i)}(t-\tau) f(\tau) d\tau \Bigg]; \end{split}$$

 $\dot{b}_{j}$  - vector density of volume forces  $(b_{j}=0)$ ; f(t) - some function;  $\rho_{j}$  - density materials  $R_{\mu}^{(i)}(t-\tau)$   $\bowtie$   $R_{\lambda}^{(i)}(t-\tau)$ - core relaxation  $\lambda_{oj}, \mu_{oj}$ - instantaneous elastic module of viscoelastic material  $\dot{u}_{j}(u_{rj}, u_{0j})$  displacement vector. Which depends on the  $r, \theta, t$ . At pressures up to 100 M Pa in the fluid motion is described satisfactorily by the wave velocity potential for the fluid particles [11].

$$\Delta \varphi_0 = \frac{1}{C_o^2} \frac{\partial^2 \varphi_0}{\partial t^2}, \qquad (2)$$

where  $C_0$  – acoustic velocity of sound in the fluid. Potential  $\varphi_0$  and the velocity vector of the fluid are related by  $V = grad\varphi_0$ . Fluid pressure  $r = R_0$  determined using the linear zed Cauchy – Lagrange  $P = -\rho_o C_0 \frac{\partial \varphi_0}{\partial t}$  -

Pressure of the fluid on the wall of the cylindrical shell and  $\rho_o$  -density of the liquid. Provided unsupported flow fluid normal

component of velocity and the layer on the surface of their contact  $r = R_0$  must be equal

$$\left. \frac{\partial \varphi_0}{\partial r} \right|_{r=R_0} = \frac{\partial u_{r2}}{\partial t} \bigg|_{r=R_0}, \qquad (3)$$

Where  $u_{r2}$  – layer of normal movement. At the contact of two bodies r = R, the equality of stresses and displacements (rigid contact condition)

$$u_{r1} = u_{r2}; \quad \sigma_{rr1} = \sigma_{rr2} \quad u_{\theta 1} = u_{\theta 2};$$
  
$$\sigma_{r\theta 1} = \sigma_{r\theta 2} \quad . \tag{4}$$



Fig. 1. Calculation scheme pipe with liquid

Note that in the case of the sliding contact surface of the soil pipe by the last equation in (2) becomes [2]:  $\sigma_{r\theta_1} = 0$ 

Let the incident plane wave propagates in the positive direction of the x-axis:  $\varphi_1^{(p)} = \varphi_A e^{(a_1x-\omega t)}, \quad \psi_1^{(p)} = 0$  under the influence of longitudinal waves (or  $\psi_1^{(p)} = \psi_A e^{(\beta_1x-\omega t)}, \quad \varphi_1^{(p)} = 0$  - when exposed to shear waves);  $\varphi_A$  and  $\psi_A$  - he amplitude of the incident waves;  $\omega$  - circular frequency of the incident waves. Expression  $\varphi^{(p)}$  (or  $\psi^{(p)}$ ) can be represented in polar coordinates, cylindrical layer  $r, \theta$  Through a series of

$$\varphi^{(p)} = \varphi_A \sum_{n=1}^{\infty} E_n i^n J_n(\alpha_1 r) \cos n \theta e^{-iwt}$$

where  $E_n = \begin{cases} 1, & n = 0 \\ 2, & n \ge 1 \end{cases}$ ,  $J_n$ - cylindrical Bessel function.

## Methods of solution.

The problem is solved in terms of potentials movements for representatives of Whimdisplacement vector in the form:

$$\overset{\mathbf{f}}{u}_{j} = grad \ \varphi_{j} + rot \overset{\mathbf{f}}{\psi}_{j}, \quad (j = 1, 2)$$

Where  $\varphi_i$  - The potential of longitudinal waves;  $\psi_{j}(\psi_{rj},\psi_{\theta j})$  - The vector potential of the transverse waves.

Basic equations of the theory of visco elastic (1) for this problem on a plane deformation can be reduced to the following equation

$$\left(\lambda_{oj} + 2\mu_{oj}\right) \nabla^{2} \varphi_{j} - \lambda_{oj} \int_{-\infty}^{t} R_{\lambda}^{(j)}(t-\tau) \nabla^{2} \varphi_{j} d\tau - 2\mu_{oj} \int_{-\infty}^{t} R_{\mu}^{(j)}(t-\tau) \nabla^{2} \varphi_{j} d\tau = \rho_{j} \frac{\partial^{2} \varphi_{j}}{\partial t^{2}}$$

$$\mu_{oj} \nabla^{2} \psi_{j}^{\mathsf{r}} - \mu_{oj} \int_{-\infty}^{t} R_{\mu}^{(j)}(t-\tau) \nabla^{2} \psi_{j} d\tau = \rho_{j} \frac{\partial^{2} \psi_{j}}{\partial t^{2}}$$

$$(5)$$

 $\nabla^{2} = \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^{2}} + \frac{\partial^{2}}{\partial \theta^{2}}$ where differential operators in cylindrical coordinates and  $v_j$  - Poisson's ratio [12].

At infinity  $r \rightarrow \infty$  potentials of longitudinal and transverse waves in j = 1 were satisfied with the Somerfield radiation condition [1]:

$$\lim_{r \to \infty} \varphi_1 = 0, \lim_{r \to \infty} (\sqrt{r})^{\kappa} \left( \frac{\partial \varphi_1}{\partial r} + i \alpha_1 \varphi \right) = 0,$$
(6)

$$\lim_{r \to \infty} \psi_1 = 0,$$
  
$$\lim_{r \to \infty} (\sqrt{r})^{\kappa} \left( \frac{\partial \psi_1}{\partial r} + i\beta_1 \psi_1 \right) = 0.$$

Equation (3) can be found in the form:

$$\varphi_{j}(r,\theta,t) = \sum_{k=1}^{\infty} q_{kj}^{(\varphi)}(r,\theta) e^{-i\omega t}; \quad \psi_{j}(r,\theta,t) = \sum_{k=1}^{\infty} q_{kj}^{(\psi)}(r,\theta) e^{-i\omega t}, \quad (7)$$

$$q_{kj}^{(\varphi)}(r,\theta) \quad \bowtie \quad q_{kj}^{(\psi)}(r,\theta) - \text{complex} \qquad \overline{\lambda}_{oj} = a_{\lambda j}(\omega) + ib_{\lambda j}(\omega), \quad \overline{\mu}_{oj} = a_{\lambda \mu}(\omega) + ib_{\mu j}(\omega)$$

Where  $q_{kj}^{(\varphi)}(r, heta)$  и  $q_{kj}^{(\psi)}(r, heta)$  – complex function, which is to solve the following equations (7)

$$\nabla^{2} q_{kj}^{(\phi)}(r,\theta) + \alpha_{j}^{2} q_{kj}^{(\phi)} = 0, \quad \nabla^{2} q_{kj}^{(\psi)}(r,\theta) + \beta_{j}^{2} q_{kj}^{(\psi)} = 0,$$
  
$$\nabla^{2} q_{k0}^{(\phi)}(r,\theta) + \alpha_{0}^{2} q_{k0}^{(\phi)} = 0, \quad j = 1,2 \quad (8)$$

2

where

here 
$$\alpha_j^2 = \frac{\rho\omega^2}{\lambda_{oj}(1-\overline{\lambda}_{oj})+2\mu_{oj}(1-\overline{\mu}_{oj})},$$
  
 $\beta_j^2 = \frac{\rho\omega^2}{\mu_{oj}(1-\overline{\mu}_{oj})}, \quad \alpha_0^2 = \frac{\omega^2}{C_0^2}$ 

$$a_{\lambda j}(\omega) = \int_{0}^{\infty} R_{\lambda j}(\tau) \sin \omega \tau \, d\tau ,$$
$$b_{\lambda j}(\omega) = \int_{0}^{\infty} R_{\mu j}(\tau) \cos \omega \tau \, d\tau .$$

The solution of equation (3) with (8) is expressed in terms of the Henkel function of the 1st and 2nd kind n-the order:

$$\varphi_1 = \sum_{n=0}^{\infty} \left[ A_n H_n^{(1)}(\alpha_1 r) + A_n' H_n^{(2)}(\alpha_1 r) \right] \cos n \,\theta e^{-iwt}$$

$$\psi_{1} = \sum_{n=0}^{\infty} \left[ B_{n} H_{n}^{(1)}(\beta_{1}r) + B_{n}' H_{n}^{(2)}(\beta_{1}r) \right] \sin n \theta e^{-iwt}$$

$$\varphi_{2} = \sum_{n=0}^{\infty} \left[ C_{n} H_{n}^{(1)}(\alpha_{2}r) + D_{n} H_{n}^{(2)}(\alpha_{2}r) \right] \cos n \theta e^{-iwt}$$
(9)

$$\psi_{2} = \sum_{n=0}^{\infty} \left[ M_{n} H_{n}^{(1)}(\beta_{2}r) + L_{n} H_{n}^{(2)}(\beta_{2}r) \right] \sin n \theta e^{-iwt}$$

$$\varphi_0 = \sum_{n=0}^{\infty} \left[ K_n J_n(\alpha_0 r) + K'_n N_n(\alpha_0 r) \right] \cos n \theta e^{-iwt}$$

Where  $A_n, A'_n, B_n, B'_n, C_n, D_n, L_n$ ,  $M_n$ ,  $K_n$  M  $K'_n$ - The expansion coefficients, which are determined by the appropriate boundary conditions;  $H_n^{(1)}(\alpha_j r)$  and  $H_n^{(2)}(\alpha_j r)$ -Respectively the Henkel function of the 1st and 2nd kind n-the order  $H_n^{(1),(2)}(\alpha r) = J_n(\alpha r) \pm iN_n(\alpha r)$ 

Solution (9) at j = 1 satisfies at infinity  $r \rightarrow \infty$ the Summerfield radiation condition (6) andisrepresentedas:

$$\varphi_1^{(r)} = \sum_{n=1}^{\infty} A_n^{(1)} H_n^{(1)}(\alpha_1^* r) \cos((m\theta) e^{-i\omega t}; \psi_1^{(r)}) = \sum_{n=1}^{\infty} C_n^{(1)} H_n^{(1)}(\beta_1^* r) \sin((n\theta) e^{-i\omega t}).$$

Solving the problem (2) satisfies the constraint condition at power factors [1], and it follows that  $K'_n = 0$ 

$$\varphi_0 = \sum_{n=0}^{\infty} K_n J_n(\alpha r) \cos n \theta e^{-iwt}$$

Full potential can be determined by applying a potential incident and reflected waves. Thus, the bias potential will be 
$$\begin{split} \phi_1 &= \varphi^{(p)} + \varphi_1^{(1)}, \\ \phi_2 &= \varphi_2, \quad \Psi_1 = \psi_1, \quad \Psi_2 = \psi_2, \quad \phi_0 = \varphi_0 \ \text{(10)} \end{split}$$

It follows that the stresses and displacements can be easily expressed in terms of bias potential [2],

$$u_{rj} = \frac{\partial \phi_{j}}{\partial r} + \frac{1}{r} \frac{\partial \psi_{j}}{\partial \theta}; \quad u_{\theta j} = \frac{1}{r} \frac{\partial \phi_{j}}{\partial \theta} - \frac{\partial \psi_{j}}{\partial r}, \quad (11)$$

$$\sigma_{rrj} = \overline{\lambda} \nabla^{2} \phi_{j} + 2 \overline{\mu}_{j} \left[ \frac{\partial^{2} \phi_{j}}{\partial r^{2}} + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi_{j}}{\partial \theta} \right) \right];$$

$$\sigma_{\theta \theta j} = \overline{\lambda} \nabla^{2} \phi_{j} + 2 \overline{\mu}_{j} \left[ \frac{1}{r} \left( \frac{\partial \phi_{j}}{\partial r} + \frac{1}{r} \frac{\partial^{2} \phi_{j}}{\partial \theta^{2}} \right) + \frac{1}{r} \left( \frac{1}{r} \frac{\partial \psi_{j}}{\partial \theta} - \frac{\partial^{2} \psi_{j}}{\partial r \partial \theta} \right) \right];$$

$$\sigma_{r\theta j} = \overline{\mu} \left\{ 2 \left[ \frac{1}{r} \frac{\partial^{2} \phi_{j}}{\partial \theta \partial r} - \frac{1}{r^{2}} \frac{\partial \phi_{j}}{\partial \theta} \right] + \left[ \frac{1}{r^{2}} \frac{\partial^{2} \psi_{j}}{\partial \theta^{2}} - r \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial \psi_{j}}{\partial r} \right) \right];$$

www.jmest.org

JMESTN42350100

Displacement and stress for the case of the layer on the compression wave  $\psi$  obtained. After substituting (10) into (11) with (9), we obtain the following expression for the displacements and stresses:

$$E_{21}^{(k)} = -\left(n^2 + n + \frac{\beta^2 r^2}{2} - \alpha^2 r^2\right) Y_n^{(k)}(\alpha r) + \alpha r Y_{n-1}^{(k)}(\alpha r) E_{42}^{(k)} = -\left(n^2 + n - \frac{\beta^2 r^2}{2}\right) Y_n^{(k)}(\beta r) + \beta r H_{n-1}^{(k)}(\beta r)$$

www.jmest.org

JMESTN42350100

$$E_{51}^{(k)} = \left[ \alpha r Y_{n-1}^{(k)}(\alpha r) - n Y_{n}^{(k)}(\alpha r) \right]$$

$$E_{52}^{(k)} = -n Y_{n}^{(k)}(\beta r)$$

$$E_{61}^{(k)} = -n Y_{n}^{(k)}(\alpha r)$$

$$E_{62}^{(k)} = \left[ n Y_{n}^{(k)}(\beta r) - \beta r Y_{n-1}^{(k)}(\beta r) \right], \quad k = 1, 2, 3, 4$$

где  
$$Y_n^{(1)} = J_n, \quad Y_n^{(2)} = N_n, \quad Y_n^{(3)} = H_n^{(1)}, \quad Y_n^{(4)} = H_n^{(2)}$$

Undetermined coefficients  $A_n^{(j)}, B_n^{(j)}, C_n^{(j)}, D_n^{(j)}$  determined from the

system of linear algebraic equations of the seventh order

$$[C]{q} = [F] \tag{13}$$

where {q}-column vector containing arbitrary constants; {F}-column vector of external loads; [C]-square matrix elements-which are expressed through the Bessel and Henkel. Equation (13) can be solved by Gaussian elimination with the release of the main element.

when  $r \rightarrow 0$ :

$$H_0^{(1),(2)}(z)\Big|_{r\to 0} = \pm \frac{2i}{\pi} \ln z - i\left(\frac{z}{2}\right)^2 \left(1 - \frac{2}{\pi} \ln z\right) + 0\left(z^4 \ln z\right),$$
  
$$H_m^{(1),(2)}(z)\Big|_{r\to 0} = \mathsf{m}\frac{i}{\pi} \left(\frac{z}{2}\right)^2 \left\{(n-1)! + n! z^2 + 0(z^4)\right\},$$

End 
$$r \rightarrow \infty$$
  
 $H_m^{(1),(2)}(z) = \left(\frac{2}{\pi z}\right)^{1/2} e^{\pm i(kz - \pi/4)},$ 

$$* u_{rj} = * u_{\ell j}$$

Henkel asymptotic formula used 1st and 2nd kind [14]. In this paper, displacement and stress is reduced in dimensionless forms

$$u_{rj}^* = \frac{u_{rj}}{i\alpha\varphi_A}; \quad u_{\theta j}^* = \frac{u_{\theta j}}{i\alpha\varphi_A}; \quad \sigma_{rrj}^* = \frac{\sigma_{rrj}}{\sigma_0}; \quad \sigma_{r\theta j}^* = \frac{\sigma_{r\theta j}}{\sigma_0}; \quad \sigma_0 = -\overline{\mu}\beta^2\varphi_A$$

## The results of calculations and conclusions

Data falling oxen stresses and displacements are defined in rows, being described by the expressions (9) - (12) in the case of hard contact. Calculations were performed by Nene computer program complex «Mat lab», series calculated with an accuracy up to  $10^{-8}$ . All expressions for the stresses and displacements are of the form:

$$(R + i \operatorname{Im})e^{-iwt} = (R^2 + \operatorname{Im}^2)^{1/2}e^{-i(wt-\gamma)}$$

As you can see the solution of the problem is expressed in terms of special functions of Bessel and Henkel functions of the 1st and 2nd kind. With the increasing number of their argument (9) - (11) converges. Therefore, on the basis of numerical experiments showed that the accuracy of 5-6 members of the ranks of the accuracy achieved  $10^{-6} - 10^{-8}$ . As relaxation kernel take a three-parameter viscoelastic core  $R(t) = \frac{Ae^{-\beta t}}{t^{1-\alpha}}$  Rizhanitsena-Koltunova [3], has a weak singularity, where  $A, \alpha, \beta$  - materials parameters [3]. We use the following parameters:  $A = 0,048; \quad \beta = 0,05; \quad \alpha = 0,1;$ 

$$C_0 = 1493 \frac{M}{c}; \quad \rho_0 = 1000 \frac{K\Gamma}{M^3};$$

$$E_{2} = 1,95 \cdot 10^{11} \frac{H}{M^{2}}; \quad \rho_{2} = 7,86 \cdot 10^{11} \frac{K\Gamma}{M^{3}};$$
$$E_{1} = 2,1 \cdot 10^{10} \frac{H}{M^{2}};$$
$$\rho_{1} = 2,0 \cdot 10^{3} \frac{K\Gamma}{M^{3}}; \quad R_{1} = 0,02R$$

Maximum voltage in a layer of liquid on the action of longitudinal and transverse harmonic waves is the radial stress. They change along with the  $\theta$ -vedena in Table 1 and 2 (with or without a liquid medium).

θ	00	45 <sup>°</sup>	90 <sup>°</sup>	135 <sup>°</sup>	180 <sup>°</sup>
$\sigma_{rr}$	0,672	0,423	0,711	0,518	1,65
Empty layer					
$\sigma_{\scriptscriptstyle rr}$	0,778	0,435	0,721	0,547	1,886
layer liquor					

Table 2. Radial stress in a layer of liquid at the impact of transverse waves

angle: $ heta$	00	45°	90 <sup>°</sup>	135 <sup>°</sup>	$180^{0}$
$\sigma_{rr}$	0,431	0,712	0,521	0,801	0,847
Empty layer					
$\sigma_{rr}$	0,483	0,914	0,637	0,825	0,886
layer liquor					

In the long-wavelength  $(\frac{D}{\lambda} > 1)$  stress distribution layer and the liquid without liquid differs to 14%, and in the short-wave (

 $\frac{D}{\lambda}\langle 1 \rangle$  in some frequency values they differ up to 40%. Accounting for the viscous properties of the material at ambient based

on longitudinal and transverse effects of harmonic waves and reduces the stresses and displacements at 10-16%. The maximum radial stress when exposed to longitudinal waves is achieved  $\theta = 90^{\circ}$  and 270°: It should be noted that the maximum radial stress at the impact of the transverse waves is achieved  $\theta$  = 45<sup>0</sup> and 135<sup>0</sup>; as well as the distribution of stress at  $\beta_1 R_1 = 0,099$  almost the same as in the static case  $(\lambda \rightarrow \infty)$  while at higher wave ( $\beta_1 R_1 = 1,5$ ) numbers of the stress distribution is significantly different from the static. The ratio of the densities  $\eta = \rho_1 / \rho_2$  has a great influence on the voltage and the bias layer. With the increasing density of the layer maximum voltage and bias layer increases. Thus, methods and algorithms for solving tasks, allow us to find the stress-strain state of cylindrical bodies under the influence of harmonic waves.

## References

 AN Guz, Kubenko V.D, M.A. Cherevenko Diffraction of elastic waves. "Science", 1978.
 308 p.

2. Pao Y.H., Mow C.C. diffraction of elastic waves and dynamic stress concentration. Number 4, Grane, Russak, 1973 694 p.

3. KOLTUNOV M.A. Creep and relaxation. - Moscow: Higher School, 1976. – 276p.

4. Datta S.K. Tensional waves in an infinite elastic solid containing a penny - shaped crack.-z. answer. Math. And Phys., 1970, 21, № 3, p.343-351

5. Mubarak Y.N., Safarov I.I. On the action of elastic waves on a cylindrical shell. Publisher

Tashkent series of Technical Sciences, 1987. № 4. p. 34-40

6. Safarov I.I. Evaluation of underground structures seysmonapryazhennogo methodology wave dynamics Collection "seism dynamics jobs and facilities" Tashkent, Science. 1988.

7. Filippov I.G. , O.A. Egorychev Unsteady oscillations and waves in the diffraction of acoustic and elastic media. . - M.: Mechanical, 1977.-304 with.

8. Safarov I.I. The interaction of waves in multilayer cylindrical layers repositories boundless elastic medium. Proceedings VII vssoyuz. Conference 'Dynamics, the basic foundations and underground structures "Dnepropetrovsk, 1989. p. 56-57

9. Safarov I.I., Dzhumaev Z.F. On blowup tunnel with strong movements of the earth. International conference on earthquake engineering. Publisher Sank - Peturburg, 2000, p. 71-78

10. Avliyakulov N.N., Safarov I.I. Modern problems of statics and dynamics of underground pipelines. Tashkent. 2007. 306 p.

 Bozorov M.B., Safarov I.I., Shokin Y.I.
 Numerical simulation of oscillations of homogeneous and inhomogeneous dissipative mechanical systems. The Novosibirsk: Publishing House. SB RAS.
 1996.189 with.

12. Rashidov T.R., Safarov I.I. and others about the two basic methods of studying the seismic stress state of underground structures under the action of seismic waves. Tashkent: DAN. Number 6, 1989. Pp. 13-17.

13. Safarov I.I. Avliyakulov N.N. Methods to improve seismic resistance of underground plastic pipe / / Uzbek Oil and Gas Journal, 2005, № 44-4.S.42.

14. Grace E., Matthews G.B. Bessellya functions and their application to physics and mechanics ke.- Publisher Moscow, 1953 - 371 p.