Relationship between New Types of Chaotic Maps

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Abstract - In this paper, we will study a new class of chaotic maps on locally compact Hausdorff spaces called topological λ-type chaotic maps and δ-type chaotic maps. The λ-type chaotic map implies chaotic map which implies δ-type chaotic map and investigate some of its properties in (X,τ^{δ}), where τ^{δ} denotes the δ–topology of a given topological space (X,τ). We have proved that every topologically λ-type transitive map is transitive which is δ-type transitive map but the converse not necessarily true, unless the space X is regular and that every λ-minimal map is minimal which is a δ- minimal map, but the converse not necessarily true. Further, the definition of topological λ-type chaos implies John Tylar definition which implies δ-type chaos definition. Relationships with some other types of chaotic maps are given.

Keywords—Topologically δ- type chaotic maps, λ- minimal systems, λ- irresolute maps, δ- dense set.

I. INTRODUCTION

In this paper, we study and introduce some new class of topological chaotic maps called topological δ-type chaotic map using δ-type transitive maps[1] and investigate some of its properties in (X,τ^{δ}), where τ^{δ} denotes the δ–topology of a given topological space (X,τ).Further, we study its relationship with λ-type chaotic map. Let A be a subset of a *topological space (X, τ). The closure and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A of a topological space (X, τ) is said to be regular open [10] (resp. preopen [11]) if A = Int(Cl(A)) (resp. A ⊆ Int(Cl(A))). A set A ⊆ X is said to be δ-open [9] if it is the union of regular open sets of a space X. The complement of a regular open (resp. δ-open) set is called regular closed (resp. δ-closed). The intersection of all δ-closed sets of (X, τ) containing A is called the δ-closure [9] of A and is denoted by* $Cl_{\delta}(A)$*.. Recall that a set S is called regular closed if* $S = Cl(Int(S))$*. A point x ε X is called a δ-cluster point [9] of S if* $S \cap U \neq \phi$ *for each regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by* $Cl_{\delta}(S)$*. A subset S is called δ-closed if δCl(S) = S. The complement of a δ-closed set is called δ-open. The family of all δ-open sets of a space X is denoted by* $\delta O(X,\tau)$*..The δ-interior of S is denoted by* $\mathrm{Int}_{\delta}(S)$ *and it is defined as follows*

$\mathrm{Int}_{\delta}(S) = \{x \in X : x \in U \subseteq \mathrm{Int}(Cl(U)) \subseteq S\}$ for some open set U of X

A subset A of a space X is called a Λ-set if it coincides with its kernel (saturated set), i.e. to the intersection of all open supersets of A. A subset A of a space X is called λ-closed [19] if A=L∩C, where L is a Λ-set and C is a closed set. The complement of a λ-closed set is called λ-open set. We denote the collection of all λ-open (resp. λ-closed) sets by λO(X) (resp. λC(X)).A point x ∈ X is called λ-cluster point of a subset A ⊂ X [20,21] if for every λ-open set U of X containing x A∩U ≠ϕ . The set of all λ-cluster points is called the λ-closure of A and is denoted by $Cl_{\lambda}(A)$. A point x ∈ X is said to be a λ-interior point of a subset A ⊂ X if there exists a λ-open set U containing x such that U ⊂ A. The set of all λ-interior points of A is said to be the λ-interior of A and is denoted by $Int_{\lambda}(A)$

We define and introduce some new class of topological transitive maps called topological δ-type transitive and study some of its properties. let as denote the collection τ^{δ} of all *δ*–sets of a space (*X; τ*), recall that *τ*=τ^{δ} if and only if every open set is closed and therefore δ-type transitive and transitive maps are coincide.

We observe that for any topological space (X, τ) the relation $^{2^{-}} \subseteq {}^{2^{¢}} \subseteq {}^{2}$ always holds. We also have $A \subseteq Cl(A) \subseteq Cl^{¢}(A) \subseteq Cl^{-}(A)$ for any subset A of X.

II. PRELIMINARIES AND DEFINITIONS

Definition 2.1. Let A be a subset of a space X. A point x ∈ A is said to be a
δ- Limit point of A if for each δ − open set U containing x, $U \cap (A-\{x\}) \neq \Phi$. The set of all δ- limit points of A is called the δ -derived set of A and is denoted by D_{δ} (A)

Definition 2.2 .[8] Let *X* be a topological space. A subset *S* of *X* is said to be regular open (respectively regular closed) if *Int.*(*cl.S*) = *S* (respectively *Cl.*(*int.S*) = *S*). A point *x* *S* is said to be a *δ*-cluster point of *S* if $U \cap S \neq ¤$, for every regular open set *U* containing *x*. The set of all *δ*-cluster point of *S* is called the *δ*-closure of *S* and is denoted by Cl_{δ} (S). If Cl_{δ} (S) = *S*, then *S* is said to be δ-closed. The complement of a *δ*-closed set is called an *δ*-open set.
For every topological space (*X, τ*), the collection of all *δ*-open sets form a topology for *X*, which is weaker than *τ* . This topology $^{2^{¢}}$ has a base consisting of all regular open sets in (*X, τ*).

Definition 2.3 A map $f:(X,^{2}) \to (Y,\pm)$ is said to be β-irresolute if $f^{-1}(V)$ is β-open in X for each β-open set V of Y (see [11])

Definition 2.4. Let $(X,^{2})$ be a topological space. A subset A is called a locally closed set (briefly LC-set) [14], if $\mathrm{A} = \mathrm{U} \cap \mathrm{F}$, where U is open and F is closed

Lemma 2.5 ([9]). Let D be a subset of X. Then:
(1) D is a δ-open set if and only if $\mathrm{Int}_{¢}(\mathrm{D}) = \mathrm{D}$.
(2) $Cl_{¢}(D^{c}) = (Int_{¢}(D))^{c}$ *and* $Int_{¢}(D^{c}) = (Cl_{¢}(D))^{c}$.
(3) $Cl(D) \subseteq Cl_{¢}(D)$ (*resp.* $Int_{¢}(D) \subseteq Int(D)$ for any subset D of X
(4) *for an open* (*resp. closed*) *subset* D *of* X, $Cl(D) = Cl_{¢}(D)$ (*resp.* $Int_{¢}(D) = Int(D)$.

Lemma 2.6 ([9]). *If X is a regular space, then:*
(1) $Cl(D) = Cl_{¢}(D)$ *for any subset* D *of* X; (2) *Every closed subset of X is δ-closed and hence for any subset D,* $Cl_{¢}(D)$ *is δ-closed.*

Definition 2.7. A map $f:(X,^{2}) \to (Y,\pm)$ is said to be *δ*-continuous [8]) if for every $A \in \pm$, $f^{-1}(A) \in ¢O(X,^{2})$ .or, equivalently, *f* is *δ*-continuous) if and only if for ever *δ*-closed set *A* of $(Y,\pm)$, $f^{-1}(A) \in ¢C(X,^{2})$.

Definition 2.8 A topological space (*X, τ*) is said to be *δ*-*T*1 [4] if for each pair of distinct points *x*, *y* of *X* there exists a *δ*-open set *A* containing *x* but not *y* and a *δ*-open set *B* containing *y* but not *x*, or equivalently, (*X, τ*) is a *δ*-*T*1-space if and only if every singleton is δ-closed ([2], Theorem 2.5).

Example 2.9. Let (X, τ) be a topological space such that X = {a; b; c; d} and
τ ={ϕ, X, {c}, {c, d},{a, b}, {a, b, c}. Clearly, δO(X, τ)={ϕ, X, {a, b}, {c, d}

Theorem 2.10 A map $f:(X,^{2}) \to (Y,\pm)$ is said to 10 δ-β irresolute [12] (resp. δ-β-continuous [13]) if $f^{-1}(V)$ is δ-β-open (resp. δ-β-open) in X for every δ-β-open (resp. open) subset V of Y.

Theorem 2.11[16] For subsets A, B of a space X, the following statements hold:

(1) $D(A) \subset D_{-}(A)$ where D(A) is the derived set of A.
(2) *If* $A \subset B$, *then* $D_{-}(A) \subset D_{-}(B)$
(3) $D_{-}(A) \cup D_{-}(B) = D_{-}(A \cup B)$ and $D_{-}(A \cap B) \subset D_{-}(A) \cap D_{-}(B)$ Note that the family $^{2^{-}}$ of θ –open sets in $(X,^{2})$ always forms a topology on X denoted θ-topology and that θ-topology coarser than τ
We observe that for any topological space (X, τ) the relation $^{2^{-}} \subseteq {}^{2^{¢}} \subseteq {}^{2}$ always holds.

Recall that a subset *S* is a Λ-set (resp. a V-set) if and only if it is an Intersection (resp. a union) of open (resp. closed) sets and that a subset *A* of a topological space X is called a *λ-open set* (or *λ*-closed) if *A* = *L* ∩*F*, where *L* is a Λ -set and *F* is closed. Complements of *λ*-closed sets will be called λ-*open*.

Definition 2.12. [21]A function $f: X \to X$ is called λ-irresolute if the inverse image of each λ-open set is a λ-open set in X.

Example 2.13[22] Let (X, τ) be a topological space such that X={a, b, c,} and τ ={ϕ, {a} ,{a, b},X}. We have λO(X, τ)={ϕ, X, {a}, {c}, {a, b}, {a, c}, {b, c}. Define the map f : X→X as follows f(c)= a, f(b)= b, f(a)= c. Then f is λ-irresolute.

Definition 2.14. A topological space $(X,^{2})$ is irreducible if every pair of nonempty open subsets of the space X has a nonempty intersection In the study of dynamics on a topological space, it is natural and convenient to break the topological space into its irreducible parts and investigate the dynamics on each part. The topological property that precludes such decomposition is called topological transitivity. In [22], Mohammed Nokhas Murad introduced the definitions of topological λ-type transitive and topologically λ-mixing maps as follows

Definition 2.15. [22] Let (X, τ) be a topological space, $f: X \to X$ be λ-irresolute map, then the map f is called λ-type transitive if for every pair of non-empty λ-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq$ ¤..

Definition 2.16.[22] Let (X, τ) be a topological space, $f: X \to X$ be λ-irresolute map, then the map f is called topologically λ-mixing if, given any nonempty λ-open subsets $U, V \subseteq X$ $\exists N \geq 1$ such that $f^n(U) \cap V \neq$ ¤ for all n>N. Clearly if *f* is topologically λ-mixing then it is also λ-transitive but not conversely

Definition 2.17[22] Two topological systems (*X, f*) and (*Y, g*) are said to be *conjugate* if there is a homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$ First of all, any property of topological systems must face the obvious question: Is it preserved under topological conjugation? That is to say, if f has property P and if we have a commutative diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & X \\ h\downarrow & & \downarrow h \\ Y & \xrightarrow[g]{} & Y \end{array}$$

Where (X, f) and (Y, g) are topological systems and h is a homeomorphism, then, is g necessarily has property P? Certainly transitivity and the existence of dense periodic points are preserved as they are purely topological conditions.

III. Δ-TRANSITIVE FUNCTIONS AND MINIMAL SYSTEMS

By a topological system (X, f) we mean a topological space X and a continuous map $f: X \to X$ A set $A \subseteq X$ is called f-invariant if $f(A) \subseteq A$. Topologically transitive and existence of a dense orbit are two notions that play an important rule in every definition of chaos.

Definition 3.1. Let $(X, ²)$ be a topological space, and $f: X \to X$ a continuous function, then f is said to be a topologically transitive function if for every pair of open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq$ ¤

A system (X, f) is called δ-*minimal* if X does not contain any non-empty, proper, δ- closed f-invariant subset[1]. In such a case we also say that the map f itself is δ-minimal. Another definition of minimal function is that if the orbit of every point x in X is dense in X then the map f is said to be minimal. Let us introduce and study an equivalent new definition.

Definition 3.2[1] (δ-minimal)Let X be a topological space and f be δ-irresolute function on X. Then (X, f) is called δ-minimal system. (or f is called δ-minimal map on X) if one of the three equivalent conditions hold:

(1) The orbit of each point of X is δ-dense in X.
(2) $Cl_¢(O_f(x)) = X$ for each x є X .
(3) Given x є X and a nonempty δ-open U in X, there exists n є N such that $f^n(x) \in U$.

Theorem 3.3.[1] For (X, f) the following statements

are equivalent:

(1) f is an δ-minimal function.
(2) If E is an δ-closed subset of X with $f(E) \subset E$, we say E is invariant. Then E=¤ or E=X.
(3) If U is a nonempty δ-open subset of X, then $\bigcup_{n=0}^{\infty} f^{-n}(U) = X$.

Proposition 3.4 [1] Let X be a δ-compact space without isolated point, if there exists a δ-dense orbit, that is there exists x_0 ∈X such that the set $O_f(x_0)$ is δ- dense then f is topologically δ-transitive .

Proof .Let x_0 be such that $O_F(x_0)$ is δ-dense. Given any pair U, V of δ-open sets, by δ-density there exists n such that $f^n(x_0) \in U$, but $O_F(x_0)$ is δ-dense implies that $O_F(f^n(x_0))$ is δ-dense, there exists m such that $f^m(f^n(x_0)) \in V$. Therefore $f^{m+n}(x_0) \in f^m(U) \cap V$ That is $f^m(U) \cap V \neq ¨$ .So f is topological δ-transitive.

Definition 3.5. A system (X, f) is called *topologically δ-mixing if* for any pair U, V of *non-empty* δ-open sets there exists $N \in \mathrm{N}$ such that for all n ≥ N we have $f^n(U) \cap V \neq ¤$ .Topologically δ-mixing conveys the idea that each δ-open set U, after iterations of f, for each δ-open set V , for all n sufficiently large, $f^n(U)$ intersects V.

Definition3.6 Two topological systems $f : X \to X$ and $g : Y \to Y$ are said to be topologically δ-conjugate if there is a δr-homeomorphism $h : X \to Y$ such that $h \circ f = g \circ h$. We will call h a topological δ- conjugacy.

Proposition 3.7 [1] if $f : X \to X$ and $g : Y \to Y$ are topologically δ-conjugate . Then :

(1) f is topologically δ- transitive if and only if g is topologically δ-transitive;

(2) f is δ-minimal if and only if g is δ-minimal;

(3) f is topologically δ-mixing if and only if g is topologically δ-mixing.

Definition 3.8 Let (X, f) be a topological system, the dynamics is obtained by iterating the map. Then, *f* is said to be topologically δ--type chaotic map on X provided that for any nonempty δ-open sets U and V in X, there is a periodic point $p \in X$ such that $U \cap O_f(p) \neq ¤$ and $V \cap O_f(p) \neq ¤$.

IV. CONCLUSION

There are the main results of the paper:

Proposition 4.1 Every λ-type chaotic map is chaotic map which is a δ-type chaotic map, but the converse not necessarily true.

Proposition 4.2 Every λ-minimal map is minimal map which is a δ-minimal map, but the converse not necessarily true.

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