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Fixed Point Theorems in Complete Metric Spaces

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Abstract- In the present paper, we prove some fixed point theorems for expansion mapping in complete metric space.

Keywords:- Fixed point, Common fixed point, expansion mapping, complete metric space.

Mathematics Subject Classification:- 47H10,54H25.

I. INTRODUCTION & PRELIMINARIES

In 1984, Wang.,Li,Gao,Iseki [13] introduced the concept of expansion mappings and proved some fixed point theorems in complete metric space. In 1992, Daffer and Kaneko [5] defined an expansion condition for a pair of mappings and proved some common fixed point theorems for two mapping in complete metric spaces.

Definition 1.1: Let X be a nonempty set and let $d: X \times X \rightarrow [0, \infty)$ be a function satisfying following conditions.

(i) $d(x, y) \geq 0$
(ii) $d(x, y) = 0 \Leftrightarrow x = y$
(iii) $d(x, y) = d(y, x)$
(iv) $d(x, y) = d(x, z) + d(z, y)$

for all $x, y, z \in X$

If d is distance function on X. Then the pair (X, d) is called metric space.

Definition 1.2: A sequence $\{x_n\}$ in metric space (X, d) is called Cauchy sequence if for given $\varepsilon > 0$ there exists $n_0 \in N$ such that $\forall\, m, n > n_0$

$\Longrightarrow d(x_n, x_m) < \varepsilon$ or $d(x_m, x_n) < \varepsilon$

$i.e. \min\{d(x_n, x_m), d(x_m, x_n)\} < \varepsilon$

Definition 1.3: A sequence $\{x_n\}$ in metric space (X, d) is convergent to $x \in X$, if

$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$

In this case x is called a limit of $\{x_n\}$ and we write $x_n \longrightarrow x$.

Definition 1.4: A metric space (X, d) is called complete if every Cauchy sequence is convergent.

Definition 1.5: Let (X, d) be a metric space and $T: X \rightarrow X$ be a mapping then T is said to be expansive mapping if for every $x, y \in X$ there exit a number $r > 1$ such that

$$d(Tx, Ty) \geq r d(x, y)$$

II. MAIN RESULT

Theorem 2.1: *Let* (X, d) *be a complete metric space and let* $T: X \to X$ *be a mapping satisfying the following condition*

$$d(T^{p+1}x, T^{p+2}y) \geq \alpha \frac{d(x,T^{p+1}x)[1+d(y,T^{p+2}y)]}{1+d(x,T^{p+2}y)} + \beta \frac{d(x,T^{p+1}x)[1+d(y,T^{p+1}x)]}{1+d(x,T^{p+1}x)} + \gamma \left[\frac{d(x,T^{p+1}x)+d(y,T^{p+1}x)}{2}\right] + \delta \left[\frac{d(x,T^{p+2}y)+d(y,T^{p+2}y)}{2}\right] \quad \ldots(3.1.1)$$

For all $x, y \in X$, $\alpha, \beta, \gamma, \delta \geq 0, \gamma + \delta > 2$

$\alpha + \beta + \gamma > 1$ *and for any non-negative integer* p.

Then T *has a unique fixed point.*

Proof: *we prove this theorem for* $p = 0$

Now putting $p = 0$ *in (3.1.1) then we have*

$$d(Tx, T^2y) \geq \alpha \frac{d(x,Tx)[1+d(y,T^2y)]}{1+d(x,T^2y)} + \beta \frac{d(x,Tx)[1+d(y,Tx)]}{1+d(x,Tx)} + \gamma \left[\frac{d(x,Tx)+d(y,Tx)}{2}\right] + \delta \left[\frac{d(x,T^2y)+d(y,T^2y)}{2}\right]$$

We define a sequence $\{x_n\} \in X$ *as follow:*

$x_0 \in X, x_0 = Tx_1, x_1 = Tx_2,$

$x_2 = Tx_3, \ldots \ldots \ldots, x_n = Tx_{n+1}$

Now consider

$$d(x_0, x_1) = d(Tx_2, T^2x_2) \geq \alpha \frac{d(x_2,Tx_2)[1+d(x_2,T^2x_2)]}{1+d(x_2,T^2x_2)} + \beta \frac{d(x_2,Tx_2)[1+d(x_2,Tx_2)]}{1+d(x_2,Tx_2)} + \gamma \left[\frac{d(x_2,Tx_2)+d(x_2,Tx_2)}{2}\right] + \delta \left[\frac{d(x_2,T^2x_2)+d(x_2,T^2x_2)}{2}\right]$$

$$\geq \alpha \frac{d(x_2,x_1)[1+d(x_2,x_0)]}{1+d(x_2,x_0)} + \beta \frac{d(x_2,x_1)[1+d(x_2,x_1)]}{1+d(x_2,x_1)} + \gamma \left[\frac{d(x_2,x_1)+d(x_2,x_1)}{2}\right] + \delta \left[\frac{d(x_2,x_0)+d(x_2,x_0)}{2}\right]$$

$$\geq (\alpha + \beta + \gamma) d(x_2, x_1) + \delta[d(x_2, x_1) - d(x_1, x_0)]$$

$$\Rightarrow d(x_2, x_1) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_0, x_1)$$

$$\Rightarrow d(x_1, x_2) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_0, x_1)$$

Similarly we have

$$d(x_2, x_3) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_1, x_2)$$

$$d(x_2, x_3) \leq \left[\frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)}\right]^2 d(x_0, x_1)$$

In general we can write

$$d(x_n, x_{n+1}) \leq \left[\frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)}\right]^n d(x_0, x_1)$$

$$d(x_n, x_{n+1}) \leq K^n d(x_0, x_1)$$

where $K = \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} < 1$

Since $0 \le K < 1$ *so for* $n \to \infty, K^n \to 0$ *we have* $d(x_{n+1}, x_n) \to 0$.

Hence $\{x_n\}$ *is a Cauchy sequence in the complete metric space* X. *So there is a point* $\xi \in X$ *such that* $\{x_n\} \to \xi$.

Now

$$d(\xi, T\xi) = d(T\xi, T^2 x_{n+2})$$

$$\ge \alpha \frac{d(\xi,T\xi)[1+d(x_{n+2},T^2x_{n+2})]}{1+d(\xi,T^2x_{n+2})}$$

$$+\beta \frac{d(\xi,T\xi)[1+d(x_{n+2},T\xi)]}{1+d(\xi,T\xi)}$$

$$+\gamma \left[\frac{d(\xi,T\xi)+d(x_{n+2},T\xi)}{2}\right]$$

$$+\delta \left[\frac{d(\xi,T^2x_{n+2})+d(x_{n+2},T^2x_{n+2})}{2}\right]$$

$$\ge \alpha \frac{d(\xi,T\xi)[1+d(x_{n+2},x_n)]}{1+d(\xi,x_n)}$$

$$+\beta \frac{d(\xi,T\xi)[1+d(x_{n+2},T\xi)]}{1+d(\xi,T\xi)}$$

$$+\gamma \left[\frac{d(\xi,T\xi)+d(x_{n+2},T\xi)}{2}\right]$$

$$+\delta \left[\frac{d(\xi,x_n)+d(x_{n+2},x_n)}{2}\right]$$

Letting $n \to \infty$ *then we have*

$$d(\xi, T\xi) \ge (\alpha + \beta + \gamma) d(\xi, T\xi)$$

$$\Longrightarrow [(\alpha + \beta + \gamma) - 1] d(\xi, T\xi) \le 0$$

This gives

$$d(\xi, T\xi) = 0 \Rightarrow T\xi = \xi.$$

Hence ξ *is a fixed point of* T.

Let η *be another point fixed of* T *then by condition (3.1.1) we have*

$$d(\xi, \eta) = d(T\xi, T^2\eta)$$

$$\ge \alpha \frac{d(\xi,T\xi)[1+d(\eta,T^2\eta)]}{1+d(\xi,T^2\eta)} + \beta \frac{d(\xi,T\xi)[1+d(\eta,T\xi)]}{1+d(\xi,T\xi)}$$

$$+\gamma \left[\frac{d(\xi,T\xi)+d(\eta,T\xi)}{2}\right] + \delta \left[\frac{d(\xi,T^2\eta)+d(\eta,T^2\eta)}{2}\right]$$

$$\ge \alpha \frac{d(\xi,\xi)[1+d(\eta,\eta)]}{1+d(\xi,\eta)} + \beta \frac{d(\xi,\xi)[1+d(\eta,\xi)]}{1+d(\xi,\xi)}$$

$$+\gamma \left[\frac{d(\xi,\xi)+d(\eta,\xi)}{2}\right] + \delta \left[\frac{d(\xi,\eta)+d(\eta,\eta)}{2}\right]$$

$$d(\xi, \eta) \ge \frac{\gamma+\delta}{2} d(\xi, \eta)$$

$$\left[\left(\frac{\gamma+\delta}{2}\right) - 1\right] d(\xi, \eta) \le 0$$

$$i.e.\, d(\xi, \eta) = 0$$

$$\Longrightarrow \xi = \eta$$

Hence fixed point of T *is unique.*

Theorem 2.2: *Let* (X, d) *be a complete metric space and let* $T: X \to X$ *be a mapping satisfying the following condition*

$$d(T^{p+1}x, T^{p+2}y)$$

$$\ge \alpha\, min\{d(x, T^{p+2}y), d(y, T^{p+1}x)\}$$

$$+\beta \left\{\frac{d(x,T^{p+1}x)+d(y,T^{p+2}y)}{2}\right\}$$

... (3.2.1)

For all $x, y \in X$, $\alpha > 1, \beta > 2$, *and for any non-negative integer* p.

Then T *has a unique fixed point.*

Proof: *we prove this theorem for* $p = 0$

Now putting $p = 0$ *in (3.2.1) then we have*

$$d(Tx, T^2y) \ge \alpha\, min\{d(x, T^2y), d(y, Tx)\}$$

$$+\beta\left\{\frac{d(x,Tx)+d(y,T^2y)}{2}\right\}$$

We define a sequence $\{x_n\} \in X$ *as follow:*

$x_n = Tx_{n+1}, n = 0,1,2, \ldots.$ *and* $x_0 \in X$.

Now consider

$$d(x_n, x_{n-1}) = d(Tx_{n+1}, T^2x_{n+1})$$

$$\geq \alpha \min\{d(x_{n+1}, T^2x_{n+1}), d(x_{n+1}, Tx_{n+1})\}$$

$$+\beta\left\{\frac{d(x_{n+1},Tx_{n+1})+d(x_{n+1},T^2x_{n+1})}{2}\right\}$$

$$\geq \alpha \min\{d(x_{n+1}, x_{n-1}), d(x_{n+1}, x_n)\}$$

$$+\beta\left\{\frac{d(x_{n+1},x_n)+d(x_{n+1},x_{n-1})}{2}\right\}$$

$$\geq \min\left\{\begin{matrix}\left(\alpha+\frac{\beta}{2}\right)d(x_{n+1},x_{n-1}) \\ +\frac{\beta}{2}d(x_{n+1},x_n), \\ \frac{\beta}{2}d(x_{n+1},x_{n-1}) \\ +\left(\alpha+\frac{\beta}{2}\right)d(x_{n+1},x_n)\end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix}\left(\alpha+\frac{\beta}{2}\right)\{d(x_{n+1},x_n)-d(x_n,x_{n-1})\} \\ +\frac{\beta}{2}d(x_{n+1},x_n), \\ \frac{\beta}{2}\{d(x_{n+1},x_n)-d(x_n,x_{n-1})\} \\ +\left(\alpha+\frac{\beta}{2}\right)d(x_{n+1},x_n)\end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix}(\alpha+\beta)d(x_{n+1},x_n) \\ -\left(\alpha+\frac{\beta}{2}\right)d(x_n,x_{n-1}), \\ (\alpha+\beta)d(x_{n+1},x_n) \\ -\frac{\beta}{2}d(x_n,x_{n-1})\end{matrix}\right\}$$

$$\geq \min\left\{\frac{(\alpha+\beta)}{1+\left(\alpha+\frac{\beta}{2}\right)}, \frac{(\alpha+\beta)}{1+\frac{\beta}{2}}\right\}d(x_{n+1},x_n)$$

$$\geq \frac{(\alpha+\beta)}{1+\left(\alpha+\frac{\beta}{2}\right)}d(x_{n+1},x_n)$$

$$\geq \frac{2(\alpha+\beta)}{2+\alpha+\beta}d(x_{n+1},x_n)$$

$$\Longrightarrow d(x_{n+1},x_n) \leq \frac{(2+2\alpha+\beta)}{2(\alpha+\beta)}d(x_n,x_{n-1})$$

$$\Longrightarrow d(x_{n+1},x_n) \leq Kd(x_n,x_{n-1})$$

Where $K = \frac{(2+2\alpha+\beta)}{2(\alpha+\beta)} < 1$

Similarly we can show that

$$d(x_n,x_{n-1}) \leq Kd(x_{n-1},x_{n-2})$$

And $d(x_{n+1},x_n) \leq K^2d(x_{n-1},x_{n-2})$

Thus $d(x_{n+1},x_n) \leq K^nd(x_1,x_0)$

Since $0 \leq K < 1$ *so for* $n \to \infty, K^n \to 0$ *we have* $d(x_{n+1},x_n) \to 0$.

Hence $\{x_n\}$ *is a Cauchy sequence in the complete metric space* X. *So there is a point* $z \in X$ *such that* $\{x_n\} \to z$.

Now

$$d(z,Tz) = d(Tz,T^2x_{n+2})$$

$$\geq \alpha \min\{d(z,T^2x_{n+2}), d(x_{n+2},Tz)\}$$

$$+\beta\left\{\frac{d(z,Tz)+d(x_{n+2},T^2x_{n+2})}{2}\right\}$$

$$\geq \alpha \min\{d(z,x_n), d(x_{n+2},Tz)\}$$

$$+\beta\left\{\frac{d(z,Tz)+d(x_{n+2},x_n)}{2}\right\}$$

Letting $n \to \infty$ *then we have*

$$d(z,Tz) \geq \frac{\beta}{2}d(z,Tz)$$

$$\left(\frac{\beta}{2}-1\right)d(z,Tz) \leq 0$$

This gives

$$d(z,Tz) = 0 \Rightarrow Tz = z.$$

Hence z is a fixed point of T.

Let w be another point of T then by condition (3.2.1) we have

$$d(z,w) = d(Tz, T^2w)$$

$$\geq \alpha \min\{d(z,T^2w), d(w,Tz)\}$$

$$+\beta\left\{\frac{d(z,Tz)+d(w,T^2w)}{2}\right\}$$

$$\geq \alpha \min\{d(z,w), d(w,z)\}$$

$$+\beta\left\{\frac{d(z,z)+d(w,w)}{2}\right\}$$

$$\geq \alpha \min\{d(z,w), d(z,w)\}$$

$$\Longrightarrow (\alpha-1)d(z,w) \leq 0$$

$$\Longrightarrow d(z,w) = 0 \qquad \text{Since } \alpha > 1$$

$$\Longrightarrow z = w$$

Hence fixed point of T is unique.

Theorem2.3: *Let (X,d) be a complete metric space and let $S,T: X \to X$ are two mappings satisfying the following condition*

$$d(S^{p+1}x, T^{p+2}y)$$

$$\geq \alpha \min\{d(x,T^{p+2}y), d(y,S^{p+1}x)\}$$

$$+\beta\left\{\frac{d(x,S^{p+1}x)+d(y,T^{p+2}y)}{2}\right\}$$

$$+\gamma\left\{\frac{d(x,T^{p+2}y)+d(y,S^{p+1}x)}{2}\right\}$$

...(3.3.1)

For all $x,y \in X$, $\alpha,\beta,\gamma > 1$,and for any non-negative integer p.

Then S,T has a unique fixed point.

Proof: *we prove this theorem for $p = 0$*

Now putting $p = 0$ in (3.3.1) then we have

$$d(Sx, T^2y) \geq \alpha \min\{d(x,T^2y), d(y,Sx)\}$$

$$+\beta\left\{\frac{d(x,Sx)+d(y,T^2y)}{2}\right\} + \gamma\left\{\frac{d(x,T^2y)+d(y,Sx)}{2}\right\}$$

Let $x_0 \in X$. We define a sequence $\{x_n\} \in X$ as follow:

$$x_0 = Tx_1, x_1 = Sx_2, \dots\dots..$$

$$x_{2n} = Tx_{2n+1}, x_{2n-1} = Sx_{2n}, \dots$$

Consider

$$d(x_{2n+1}, x_{2n}) = d(Sx_{2n+2}, T^2x_{2n+2})$$

$$\geq \alpha \min\left\{\begin{matrix} d(x_{2n+2}, T^2x_{2n+2}), \\ d(x_{2n+2}, Sx_{2n+2}) \end{matrix}\right\}$$

$$+\beta\left\{\frac{d(x_{2n+2},Sx_{2n+2})+d(x_{2n+2},T^2x_{2n+2})}{2}\right\}$$

$$+\gamma\left\{\frac{d(x_{2n+2},T^2x_{2n+2})+d(x_{2n+2},Sx_{2n+2})}{2}\right\}$$

$$\geq \alpha \min\{d(x_{2n+2}, x_{2n}), d(x_{2n+2}, x_{2n+1})\}$$

$$+\beta\left\{\frac{d(x_{2n+2},x_{2n+1})+d(x_{2n+2},x_{2n})}{2}\right\}$$

$$+\gamma\left\{\frac{d(x_{2n+2},x_{2n})+d(x_{2n+2},x_{2n+1})}{2}\right\}$$

$$\geq \min\left\{\begin{matrix} \left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)d(x_{2n+2},x_{2n}) \\ +\left(\frac{\beta+\gamma}{2}\right)d(x_{2n+2},x_{2n+1}), \\ \left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)d(x_{2n+2},x_{2n+1}) \\ +\left(\frac{\beta+\gamma}{2}\right)d(x_{2n+2},x_{2n}) \end{matrix}\right\}$$

$$\geq \min\left\{\begin{matrix} \left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)\{d(x_{2n+2},x_{2n+1})-d(x_{2n+1},x_{2n})\} \\ +\left(\frac{\beta+\gamma}{2}\right)d(x_{2n+2},x_{2n+1}), \\ \left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)d(x_{2n+2},x_{2n+1}) \\ +\left(\frac{\beta+\gamma}{2}\right)\{d(x_{2n+2},x_{2n+1})-d(x_{2n+1},x_{2n})\} \end{matrix}\right\}$$

$$\geq min\begin{Bmatrix}(\alpha+\beta+\gamma)d(x_{2n+2},x_{2n+1})\\ -\left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)d(x_{2n+1},x_{2n}),\\ (\alpha+\beta+\gamma)d(x_{2n+2}\ ,x_{2n+1})\\ -\left(\frac{\beta+\gamma}{2}\right)d(x_{2n+1},x_{2n})\end{Bmatrix}$$

$$\geq\ min\left\{\frac{(\alpha+\beta+\gamma)}{1+\left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)},\frac{(\alpha+\beta+\gamma)}{1+\left(\frac{\beta+\gamma}{2}\right)}\right\}d(x_{2n+2},x_{2n+1})$$

$$d(x_{2n+1},x_{2n})\geq\frac{(\alpha+\beta+\gamma)}{1+\left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)}d(x_{2n+2},x_{2n+1})$$

$$\Longrightarrow d(x_{2n+2},x_{2n+1})$$

$$\leq\frac{(2+2\alpha+\beta+\gamma)}{2(\alpha+\beta+\gamma)}d(x_{2n+1},x_{2n})$$

$$\leq\frac{(2+2\alpha+\beta+\gamma)}{2(\alpha+\beta+\gamma)}d(x_{2n+1},x_{2n})$$

$$\Longrightarrow d(x_{2n+2},x_{2n+1})\leq Kd(x_{2n+1},x_{2n})$$

where $K=\frac{(2+2\alpha+\beta+\gamma)}{2(\alpha+\beta+\gamma)}<1$

Similarly,

$$d(x_{2n+1},x_{2n})\leq Kd(x_{2n},x_{2n-1})$$

And

$$d(x_{2n+2},x_{2n+1})\leq K^2d(x_{2n},x_{2n-1})$$

Continue in this way we get

$$d(x_{2n+2},x_{2n+1})\leq K^n d(x_1,x_0)$$

Since $0\leq K<1$ *so for* $n\to\infty, K^n\to 0$ *we have* $d(x_{2n+2},x_{2n+1})\to 0$.

Hence $\{x_n\}$ *is a Cauchy sequence in the complete metric space* X. *So there is a point* $z\in X$ *such that* $\{x_n\}\to z$.

Now we will show that z *is a common fixed point of* S *and* T.

$$d(z,Sz)=d(x_n,Sz)=d(Sz,T^2x_{n+2})$$

$$\geq\ \alpha\, min\{d(z,T^2x_{n+2}),d(x_{n+2},Sz)\}$$

$$+\beta\left\{\frac{d(z,Sz)+d(x_{n+2},T^2x_{n+2})}{2}\right\}$$

$$+\gamma\left\{\frac{d(z,T^2x_{n+2})+d(x_{n+2},Sz)}{2}\right\}$$

$$\geq\ \alpha\, min\{d(z,x_n),d(x_{n+2},Sz)\}$$

$$+\beta\left\{\frac{d(z,Sz)+d(x_{n+2},x_n)}{2}\right\}$$

$$+\gamma\left\{\frac{d(z,x_n)+d(x_{n+2},Sz)}{2}\right\}$$

$$\Longrightarrow d(z,Sz)\geq\frac{\beta+\gamma}{2}d(z,Sz)$$

$$\Longrightarrow\left(\frac{\beta+\gamma}{2}-1\right)d(z,Sz)\leq 0$$

This gives $d(z,Sz)=0\Longrightarrow Sz=z$.

Hence z *is a fixed point of* S.

Similarly we can show that z *is a fixed point of* T.

Hence z *is a common fixed point of* S & T.

Let u,v *be a common fixed point of* S *and* T *then*

$$d(u,v)=d(Su,Tv)=d(Su,T^2v)$$

$$\geq\ \alpha\, min\{d(u,T^2v),d(v,Su)\}$$

$$+\beta\left\{\frac{d(u,Su)+d(v,T^2v)}{2}\right\}$$

$$+\gamma\left\{\frac{d(u,T^2v)+d(v,Su)}{2}\right\}$$

$$\geq\ \alpha\, min\{d(u,v),d(v,u)\}$$

$$+\beta\left\{\frac{d(u,u)+d(v,v)}{2}\right\}$$

$$+\gamma\left\{\frac{d(u,v)+d(v,u)}{2}\right\}$$

$$\Rightarrow d(u,v) \geq (\alpha+\gamma)d(u,v)$$

$$\Rightarrow (\alpha+\gamma-1)d(u,v) \leq 0$$

$$\Rightarrow d(u,v) = 0$$

$$\Rightarrow u = v$$

Hence common fixed point of S *and* T *is unique.*

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