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A State Variable Model for Considering the Power Inductor DC Resistance on the Open Loop Performance of a Buck- Boost DC to DC Converter

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***Abstract*—**This paper shows a dc and small- signal circuit model for the PWM buck- boost converter with the equivalent series resistance of the inductor. The dc voltage transfer function and the efficiency of the converter are derived from the dc model. Small- signal open- loop characteristics are derived from the small- signal model based on a state variable model. A design example proves the performance of the circuit and verification of the model.

Keywords*—Dc to dc converter; buck- boost; small signal analysis*

I. INTRODUCTION

Many papers about small signal analysis of a DC to DC Buck- Boost converter can be found in literature that include parasitic resistances in inductor and capacitors and voltage drop in power switch and diode [1]- [4]. Usually the signal model representation is obtained from an equivalent circuit [4], or in other cases, using a state variable model [3]. Any model can be calculated based on the continuous conduction mode or a non- continuous conduction mode but have different results because one has control over the duty cycle and the other has control over the frequency. The objectives of this paper are: To obtain a dc and small- signal linear circuit models of a PWM buck – boost dc to dc converter, taking into account parasitic resistance of the inductor; to derive the dc voltage transfer function and efficiency; to derive small- signal open- loop input- to- output transfer function using a state variable model; and to demonstrate, by a design consideration, the performance of a real circuit.

II. WORK DESCRIPTION

A. State Variable Model

In a non-ideal Buck- Boost DC-to-DC converter it is necessary to consider power losses from parasitic resistance of the inductor, parasitic resistance of capacitors and power losses in semiconductor switch and diode. Figure 1 shows a non- ideal Buck- Boost DC-to-DC converter with voltage drops in switch and diode and

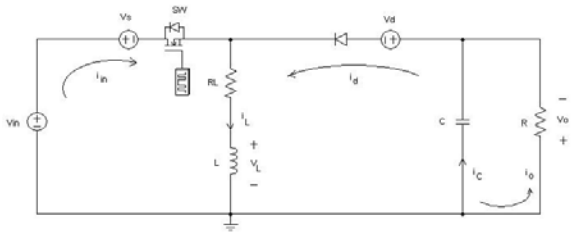

Figure 1. Circuit of Buck- Boost converter with inductance parasitic resistance and voltage drops.

parasitic resistance in inductor.

Voltage drop in switch and diode can be neglected in the analysis of the circuit if the voltage input Vin is greater than the voltage drop in switch, Vs and voltage drop in diode, Vd. Also, the parasitic resistance in series with the capacitor can be reduced by some parallel capacitors. Neglecting voltage drop in switch and diode and parasitic resistance in capacitor, when switch is ON, the following equations represent the behavior of the circuit.

$$-V_{in} + i_L R_L + L\frac{di_L}{dt} = 0 \quad (1)$$

$$-\frac{V_o}{R} = C\frac{dV_o}{dt} \quad (2)$$

And, when switch is OFF,

$$V_o + i_L R_L + L\frac{di_L}{dt} = 0 \quad (3)$$

$$i_L - \frac{V_o}{R} = C\frac{dV_o}{dt} \quad (4)$$

The state matrixes of these equations are: When switch is ON,

$$\frac{d}{dt}\begin{bmatrix} i_L \\ V_o \end{bmatrix} = \begin{bmatrix} -R_L/L & 0 \\ 0 & -1/RC \end{bmatrix}\begin{bmatrix} i_L \\ V_o \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_{in} \quad (5)$$

And when switch is OFF,

$$\frac{d}{dt}\begin{bmatrix} i_L \\ V_o \end{bmatrix} = \begin{bmatrix} -R_L/L & -1/L \\ 1/C & -1/RC \end{bmatrix}\begin{bmatrix} i_L \\ V_o \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_{in} \quad (6)$$

Adding both states, for $t_{ON} = DT$ and $t_{OFF} = (1-D)T$,

$$\frac{d}{dt}\begin{bmatrix} i_L \\ V_o \end{bmatrix} = \begin{bmatrix} -R_L/L & -(1-D)/L \\ (1-D)/C & -1/RC \end{bmatrix}\begin{bmatrix} i_L \\ V_o \end{bmatrix} + \begin{bmatrix} D/L \\ 0 \end{bmatrix} V_{in} \quad (7)$$

Using Laplace to solve this first order equation, the voltage input- to- output transfer function is,

$$G_v(s) = \frac{V_o}{V_{in}} = \frac{(1-D)\times D}{LC} \frac{1}{s^2 + \left(\frac{1}{RC} + \frac{R\propto}{L}\right)s + \frac{1}{LC}((1-D)^2 + \propto)} \quad (8)$$

Where $\propto = \frac{R_L}{R}$. The corner frequency is,

$$f_o = \frac{1}{2\pi}\sqrt{\frac{(1-D)^2+\alpha}{LC}} \quad (9)$$

In steady state the transfer function is,

$$M_{VDC} = \frac{V_o}{V_{in}} = \frac{D}{(1-D) + \frac{\propto}{(1-D)}} \quad (10)$$

Maximum gain occurs when,

$$\frac{\partial}{\partial D}\left(\frac{V_o}{V_{in}}\right) = \frac{(1-D)^2 - 2\propto D + \propto}{((1-D)^2 + \propto)^2} = 0 \quad (11)$$

That is,

$$D = (1+\propto) - \sqrt{\propto (1+\propto)} \quad (12)$$

Figure 2 shows the maximum voltage gain of the circuit for values of α between 0.01 and 0.2. As seen, lower values of α give bigger values of voltage gain and the circuit get closer to an ideal circuit.

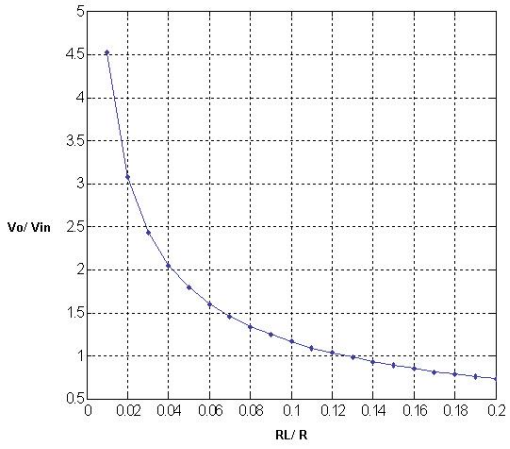

Figure 2. Gain vs. RL/R for a non ideal Buck- Boost DC-to-DC converter.

B. DC Model

Considering this Buck- Boost DC-to-DC converter operates in continuous conduction mode it is necessary to make sure the lowest inductor current is above cero:

$$I_{min} = I_L - \frac{\Delta i_L}{2} > 0 \quad (13)$$

If this condition is not considered, the inductor current will have times with zero current and the circuit will work in non-continuous mode. Here $\Delta i_L = I_{max} - I_{min}$, the peak-to-peak current of the inductor, and I_L is the average inductor current. The average inductor current is,

$$I_L = \frac{I_o}{(1-D)} = \frac{V_o}{R(1-D)} \quad (14)$$

When switch is open,

$$\Delta i_L = \frac{V_o(1-D+\propto)}{Lf} \quad (15)$$

Then,

$$L > \frac{R(1-D)^2 + R_L(1-D)}{2f} \quad (16)$$

The input current is,

$$I_{in} = I_L D \quad (17)$$

From this equation, the DC current transfer function is,

$$M_{IDC} = \frac{I_o}{I_{in}} = \frac{(1-D)}{D} \quad (18)$$

Efficiency is calculated from the equation:

$$\eta = \frac{V_o I_o}{V_{in} I_{in}} = M_{VDC} M_{IDC} = \frac{1}{1 + \frac{\alpha}{(1-D)^2}} \quad (19)$$

C. Circuit Experimentation

For validating these equations a circuit with some practical characteristics is designed and simulated. This circuit is a 1,000 watts Buck- Boost DC-to-DC converter with an input voltage Vin= 170 volts, output voltage Vo= 230 volts, frequency f= 50 khz and 5% of voltage ripple. α can go up to 0.09, as seen on figure 2. Choosing α= 0.05,

$$2.35D^2 - 3.7D + 1.42 = 0 \quad (20)$$

With this equation duty cycle could be 0.6594 or 0.9156. As the figure 3 shows, small parasitic resistance in inductance produces more voltage gain, but a maximum voltage gain does not mean a maximum efficiency. So, in order to have a good efficiency – gain relationship, it is better to work in values of D before the maximum peak of voltage gain, that is, in the left side of the curve. Also, as seen on Vo/Vin, the voltage gain curve has less slope with D between zero and D on the maximum gain than in the rest of the curve, which means that a significant change in voltage gain occurs with D varying between D at the maximum gain and one, where the system becomes unstable or more difficult to control.

In the same figure 3 it is shown the efficiency of the circuit which achieves around 70% for D=0.6594 and close to 10% with D= 0.9156. The values of resistances are R= 52.9 for the load and $R_L = 2.645$ of the permitted parasitic resistance of the inductor, so that,

$$L > 70.4uH \approx 80uH \quad (21)$$

To calculate the capacitor, an equation derived from an ideal circuit analysis, most seen on any power electronics books, can be used, since parasitic resistance of the inductor does not interfere with the ripple voltage at the output of the circuit.

$$C = \frac{D}{Rf(\frac{\Delta V_o}{V_o})} = 5uf \qquad (22)$$

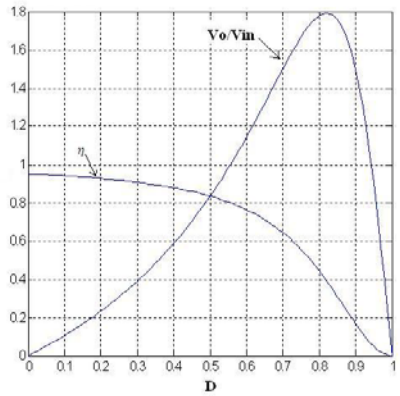

Figure 3. Vo/ Vin and Efficiency Vs. D in a non- ideal Buck- Boost DC to Dc converter.

Simulation of the circuit, using PSIM, is shown in figure 4 (with the ripple output voltage). In the same figure 4 it is shown a simulation in Matlab of the step response of the transfer function of the circuit (continuous line). Both simulations curves are superimposed to demonstrate the same response and the validation of the equations with respect to simulations.

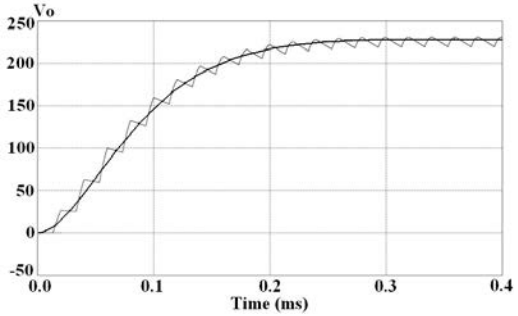

Figure 4. Comparison of circuit simulation and step response of transfer function of a non- ideal Buck- Boost DC to DC converter.

CONCLUSIONS

This analysis has concentrated on finding the ac model of dc to dc buck- boost converter, only taking into account the parasitic resistance of the inductor. This is because in large voltage conversion, voltage drop in semiconductor switch and diode can be neglected.

It is likely to point out that a complete ac analysis of the circuit requires obtaining the ac model, not only of the voltage gain, but impedance gain also.

A buck- boost dc-to-dc converter has been designed for verification of the performance of the circuit and comparison between the model and circuit simulation. As shown, the voltage output of the circuit close follows the equation obtained as a model. Also, the implication of the value of the parasitic resistance of the inductor with respect to the value of the load is shown in a curve for maximum gain, maximizing the gain of the circuit with the duty cycle as the variable to be controlled.

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